**1** $X$ is a topological space. Let $\{f_n : X \longrightarrow \mathbb{R}\}$ be a sequence of continuous real-valued functions.

(a) Define what it means for $\{f_n : X \longrightarrow \mathbb{R}\}$ to converge *uniformly* to the function $f : X \longrightarrow \mathbb{R}$.

(b) Prove that the limit of a uniformly convergent sequence of continuous functions is itself continuous.

**2** Let $f : X \longrightarrow Y$ be a continuous surjection, where $X$ is compact and $Y$ Hausdorff. Assume that $Y$ is connected and that $f^{-1}(y)$ is connected for each $y \in Y$. Prove that $X$ must be connected.

**3** Let $f : [0,1] \longrightarrow [0,1]$ be a continuous function such that $f(0) = 0$ and

$$|f(x) - f(y)| \geq |x - y|$$

for all $x, y \in [0,1]$. Prove that $f(t) = t$ for all $t \in [0,1]$.

**4** Let $p : X \longrightarrow Y$ be a quotient map and let $B \subset Y$ be a set for which $p^{-1}(y)$ is a singleton for all $y \in Y - B$. Prove that if $A = p^{-1}(B)$ is a retract of $X$, then $B$ is a retract of $Y$.

**5** A continuous function $f : X \longrightarrow Y$ is said to be proper if $f^{-1}(C)$ is compact for all compact sets $C \subset Y$.

(a) Give an example of a continuous function which is *not* proper.

(b) Let $f : X \longrightarrow Y$ and $g : Y \longrightarrow Z$ be continuous functions for which the composition $g \circ f$ is proper. Prove that $f$ must be proper provided that $Y$ is Hausdorff.

**6** Prove that if $f : S^1 \longrightarrow \mathbb{R}^1$ is a continuous function such that $f(-x) = -f(x)$ for all unit vectors $x$ in the circle $S^1$, then $f(x_0) = 0$ for some unit vector $x_0$.

**7** Let $p : E \longrightarrow B$ be a covering map, where $B$ is locally connected. Assume that there exists a continuous function $f : B \longrightarrow E$ for which $p(f(b)) = b$ for all $b \in B$. Prove that $f(B)$ must be an open set.

**8** Let $T = S^1 \times S^1$ be the torus.

(a) Let $j : S^1 \times \{(1,0)\} \longrightarrow T$ be the inclusion and use your knowledge of $\pi_1(T)$ to identify the quotient group $\pi_1(T)/\operatorname{im}(j_*)$.

(b) Now use the corollary of the Seifert-van Kampen Theorem to compute $\pi_1(X)$ where $X = T \cup B$ and $B$ is a copy of $B^2$ for which

$$T \cap B = S^1 \times \{(1,0)\} = \operatorname{Bd}(B).$$