DEPARTMENT OF MATHEMATICS

TOPOLOGY PRELIMINARY EXAMINATION
JUNE 5, 2019

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1 Let $\mathbb{R}$ be the real line with the standard topology.

(a) Prove or disprove that $\mathbb{R}/(0,1)$ with the quotient topology is homeomorphic to $\mathbb{R}$.

(b) Prove or disprove that $\mathbb{R}/[0,1]$ with the quotient topology is homeomorphic to $\mathbb{R}$.

2 Let $x \in X$ and $y \in Y$. Furthermore, let C_x be the connected component of x in X, and similarly let C_y be the connected component of y in Y. Show that $C_x \times C_y$ is the connected component of (x,y) in $X \times Y$.

3 Let G be a topological group that is T_0. (Recall that a space is T_0 if, for each pair of distinct points, at least one of the points has a neighborhood not containing the other point.)

(a) Show that points are closed in G .

(b) Use the map $\varphi : G \times G \longrightarrow G$, defined by $\varphi(g,h) = gh^{-1}$, to show that G is Hausdorff.

4 Let $f\colon X \longrightarrow Y$ be a closed map for which $f^{-1}(y)$ is compact for all $y \in Y$. Show that if X is Hausdorff, then the subspace $f(X)$ of Y is also Hausdorff.

5 Show that every map $f : \mathbb{RP}^n \longrightarrow S^1$ is homotopic to a constant map.

6 Let X be the space obtained from the torus $T = S^1 \times S^1$ by attaching a Mobius band M using a homeomorphism from the boundary circle of M to $S^1 \times \{x_0\} \subset T$. Compute $\pi_1(X, x_0)$.

7 Let K be the Klein bottle and let $T = S^1 \times S^1$ be the torus. Can there exist a covering map $K \longrightarrow T$? Why or why not?

8 Let X be path-connected, locally path-connected, and semilocally simply-connected. Assume that $p\colon X \longrightarrow X$ is a non-injective covering map. Prove that $\pi_1(X, x_0)$ is infinite for any choice of basepoint $x_0 \in X$.