

Math 225 - Practice Midterm 2

1. True or False?

(a) The set of vectors $\left\{ \begin{pmatrix} 1 \\ 0 \\ 2 \end{pmatrix}, \begin{pmatrix} 0 \\ -1 \\ 1 \end{pmatrix}, \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} \right\}$ is a basis for \mathbb{R}^3 .

(b) The set of vectors $\left\{ \begin{pmatrix} 1 \\ 0 \\ 2 \end{pmatrix}, \begin{pmatrix} 0 \\ -1 \\ 1 \end{pmatrix}, \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} \right\}$ is a basis for \mathbb{R}^3 .

(c) If U is an echelon form for the matrix A , then the pivot columns of U form a basis for $C(A)$.

(d) If U is an echelon form for the matrix A , then the nonzero rows of U form a basis for $R(A)$.

(e) If A is an $n \times n$ matrix, then $\det(-A) = -\det(A)$.

2. (a) Use Cramer's Rule to solve the matrix equation $\begin{pmatrix} 5 & 1 \\ 7 & 3 \end{pmatrix} \mathbf{x} = \begin{pmatrix} 13 \\ 15 \end{pmatrix}$.

(b) Use Cramer's Rule to find the entry in position $(1,2)$ of the *inverse* of $A = \begin{pmatrix} 2 & 1 & 5 \\ -1 & 6 & 2 \\ 1 & 3 & -2 \end{pmatrix}$.

3. (a) For which value of c does the equation $2x - y + 3z = c$ define a subspace of \mathbb{R}^3 ?

(b) Find a basis for this subspace of \mathbb{R}^3 . What is the dimension of this subspace?

4. Consider the bases

$$\mathcal{B} = \left\{ \begin{pmatrix} 1 \\ 4 \end{pmatrix}, \begin{pmatrix} 2 \\ 2 \end{pmatrix} \right\}, \quad \mathcal{C} = \left\{ \begin{pmatrix} 2 \\ -3 \end{pmatrix}, \begin{pmatrix} 1 \\ 5 \end{pmatrix} \right\}$$

for \mathbb{R}^2 .

(a) Find the change-of-basis matrix ${}_{\mathcal{C}}P_{\mathcal{B}}$.

(b) Let $\mathbf{v} = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$. Find $[\mathbf{v}]_{\mathcal{B}}$ and $[\mathbf{v}]_{\mathcal{C}}$.

5. Find all eigenvalues and a basis for each eigenspace for the following matrices. Use this to diagonalize these matrices.

(a) $A = \begin{pmatrix} 2 & 2 \\ 3 & 1 \end{pmatrix}$.

(b) $B = \begin{pmatrix} 4 & 4 & -2 \\ 0 & 0 & 0 \\ 4 & 4 & -2 \end{pmatrix}$