Thursday, October 27 $\quad ** \quad$ *Matrix algebra & Invertible matrices*

1. Consider the following matrices.

$$A = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \end{pmatrix}, B = \begin{pmatrix} 1 & 0 \\ 0 & 1 \\ 0 & 0 \end{pmatrix}, C = \begin{pmatrix} 1 & 1 \\ 2 & 2 \end{pmatrix}, D = \begin{pmatrix} 4 & -2 \\ 2 & -1 \end{pmatrix}.$$

   (a) Compute the matrics AB and BA.

   (b) Compute the matrices CD and DC.

2. (Diagonal matrices) A square matrix is called a *diagonal matrix* if all entries off of the main diagonal are 0.

   (a) Let A be any $n \times 2$ matrix and let $B = \begin{pmatrix} 2 & 0 \\ 0 & 3 \end{pmatrix}$. Describe the columns of the matrix AB in terms of the columns of A.

   (b) Generalize part (a) to describe the columns of AB if A is $n \times k$ and B is the diagonal matrix $\begin{pmatrix} b_1 & 0 & 0 \\ 0 & \ddots & 0 \\ 0 & 0 & b_k \end{pmatrix}$.

   (c) Let A be any $2 \times n$ matrix and let $B = \begin{pmatrix} 2 & 0 \\ 0 & 3 \end{pmatrix}$. Describe the rows of the matrix BA in terms of the rows of A.

   (d) Generalize part (c) to describe the rows of BA if A is $n \times k$ and B is the diagonal matrix $\begin{pmatrix} b_1 & 0 & 0 \\ 0 & \ddots & 0 \\ 0 & 0 & b_n \end{pmatrix}$.

3. (Scalar matrices) A diagonal matrix in which all of the diagonal entries are the *same* is called a scalar matrix. Use the previous problem to show that if A is any $n \times n$ matrix and D is an $n \times n$ scalar matrix, then $AD = DA$. What is another description of the matrix AD?

4. One of the following three matrices has an inverse. Which one?

$$A = \begin{pmatrix} 1 & 3 & -2 \\ 1 & 1 & -2 \\ -2 & 7 & 4 \end{pmatrix} \qquad B = \begin{pmatrix} 1 & 3 & -2 \\ 0 & 1 & -1 \\ -4 & 0 & 6 \end{pmatrix} \qquad C = \begin{pmatrix} 2 & 0 & -3 \\ 1 & 6 & -2 \\ -4 & 0 & 6 \end{pmatrix}.$$

   Hint: You shouldn't need to do any row reducing to eliminate two possiblities.