Thursday, November 17 $\quad **\quad$ *Change of basis & eigenvectors*

1. Recall that $\mathbb{P}_2$ denotes the vector space of polynomials of degree at most 2. Consider the bases
$$\mathscr{B} = \{1,\ 2t,\ -2+4t^2\}, \qquad \mathscr{C} = \{1,\ 1-t,\ 2-4t+t^2\}$$
for $\mathbb{P}_2$. Find the change-of-basis matrix ${}_{\mathscr{C}\leftarrow}P_{\mathscr{B}}$.

2. Let $\mathscr{C}$ be the basis for $\mathbb{R}^3$ given by
$$\mathscr{C} = \left\{ \begin{pmatrix} 1 \\ 2 \\ -2 \end{pmatrix}, \begin{pmatrix} 0 \\ -1 \\ -2 \end{pmatrix}, \begin{pmatrix} 3 \\ 4 \\ -8 \end{pmatrix} \right\}.$$
   (a) Find the change-of-basis matrix ${}_{\mathscr{C}\leftarrow}P_{\mathscr{E}}$, where $\mathscr{E}$ is the standard basis.

   (b) Use this to write the vector $\mathbf{v} = \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix}$ in the basis $\mathscr{C}$. That is, find $[\mathbf{v}]_{\mathscr{C}}$.

3. Find all eigenvalues and a basis for each eigenspace for the following matrices.

   (a) $A = \begin{pmatrix} 2 & 2 \\ -1 & 2 \end{pmatrix}$.

   (b) $B = \begin{pmatrix} 3 & 4 & 0 \\ 2 & 3 & 0 \\ 1 & -5 & 0 \end{pmatrix}$.

   (c) $C = \begin{pmatrix} 1 & 2 & 0 \\ 0 & 1 & 2 \\ 0 & 0 & 1 \end{pmatrix}$.