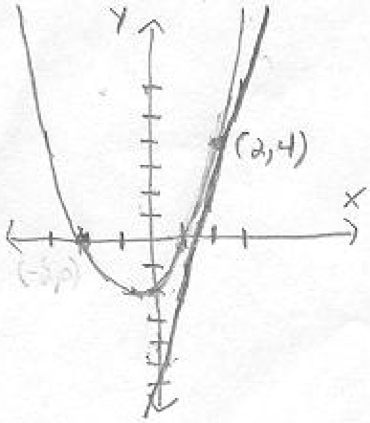
Worksheet 2 Solutions: Thursday, August 25

① $f(x) = x^2 + x - 2$



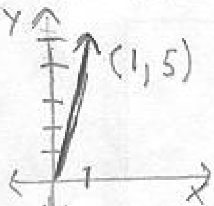

(b) $f'(x) = 2x + 1$

$m = f'(2) = 5$

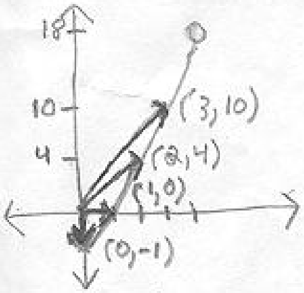
$y - y_0 = m(x - x_0)$

$y = m(x - x_0) + f(x_0) = 5(x-2) + 4 = 5x - 6$

(c)



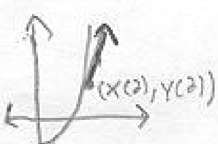

② $\begin{cases} x(t) = t \\ y(t) = t^2 + t - 2 \end{cases}$ $\quad 0 \le t < 4$




(a) This is the graph from 1(a) restricted to the interval $[0,4)$

(c) $(x'(t), y'(t)) = (1, 2t+1)$

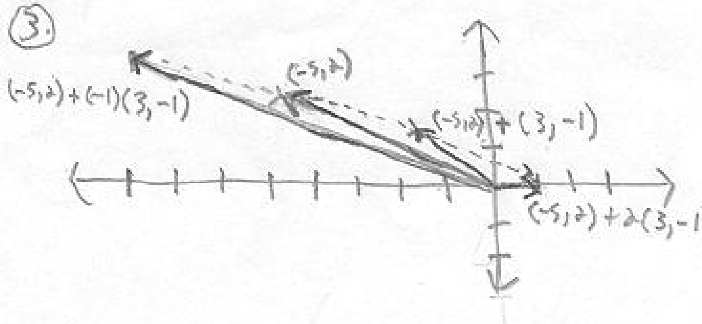
$(x'(2), y'(2)) = (1, 5)$




This vector represents the velocity (speed and direction) of the curve at time $t = 2$; it is parallel to the line tangent to the curve at $(x(2), y(2))$

(d) The speed is the magnitude of the velocity vector $(x'(2), y'(2)) = (1,5)$ from (c), **which is $\sqrt{1^2 + 5^2} = \sqrt{26}$**

③



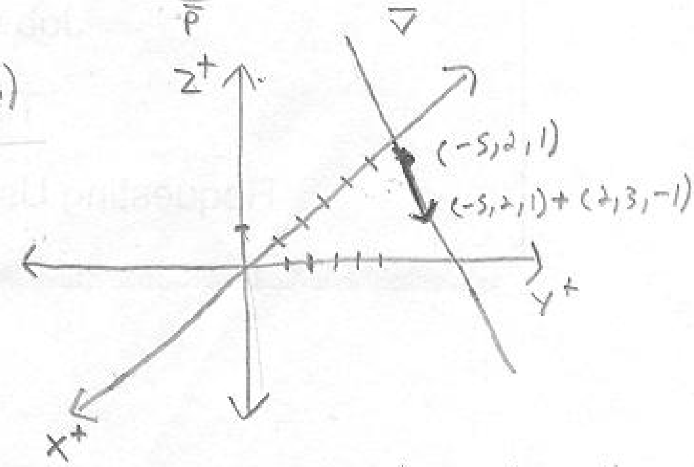

(e) $(-5,2) + t(3,-1)$ is the line in the direction $(3,-1)$ passing through $(-5,2)$

④ $\bar{\ell}(t) = (-5 + 2t, 2 + 3t, 1 - t)$

(a) $\bar{\ell}(t) = (-5, 2, 1) + (2t, 3t, -t)$

$= \underbrace{(-5, 2, 1)}_{\bar{p}} + t\underbrace{(2, 3, -1)}_{\bar{v}}$

(b)




This equation describes the line in direction $(2,3,-1)$ going through $(-5,2,1)$

(c) it gives the direction of the line

⑤ $\bar{a} = (-\sqrt{3}, 0, -1, 0)$ $\quad \bar{b} = (1, 1, 0, 1)$

(a) $d = \sqrt{(-\sqrt{3}-1)^2 + (0-1)^2 + (-1-0)^2 + (0-1)^2}$

$= \sqrt{3 + 2\sqrt{3} + 1 + 1 + 1 + 1} = \sqrt{7 + 2\sqrt{3}}$

(b) $\cos\theta = \dfrac{\bar{a} \cdot \bar{b}}{|\bar{a}||\bar{b}|} = \dfrac{-\sqrt{3} + 0 + 0 + 0}{\sqrt{3+0+1+0}\ \sqrt{1+1+0+1}}$

$= \dfrac{-\sqrt{3}}{2\sqrt{3}} = -\dfrac{1}{2}$

Thus $\theta = \cos^{-1}(-\frac{1}{2}) = \dfrac{2\pi}{3}$