Lecture 1

Aug. 22
2011

Math 241: Calculus III

You already understand

- continuity
- derivatives
- integrals

for real-valued functions of a single variable.
(single-valued)

In this course, we will study more generally functions $f: \mathbb{R}^n \longrightarrow \mathbb{R}^k$

n inputs k outputs

Examples 1) a curve in the plane $(\mathbb{R}^2)$

- the circle $f: \mathbb{R} \longrightarrow \mathbb{R}^2$

$$f(t) = (\cos t, \sin t)$$

We say f "parametrizes" the circle

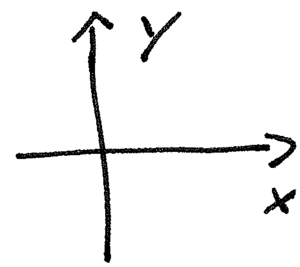
2) an "elevation" function

Let (x, y) describe position on a map.

Let $g(x,y)$ = the elevation at position (x,y)

What does it mean to say f & g are continuous? or differentiable? integrable?

Ch. 12, Sect. 12.1



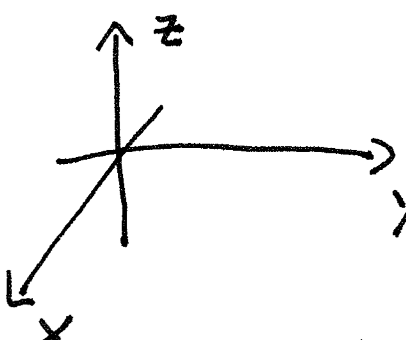
$\mathbb{R}^2$ the cartesian plane

2-dimensional, point described by (x,y) two coordinates.

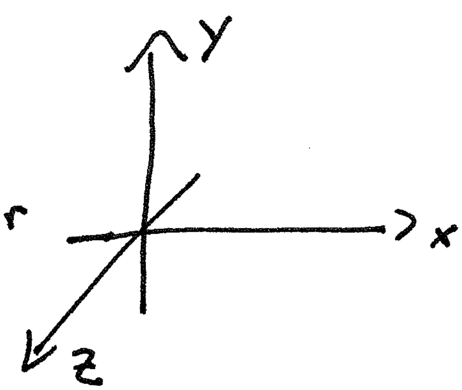
$\mathbb{R}^3$ 3-dimensional space

3 axes, point described by (x,y,z) three coordinates.

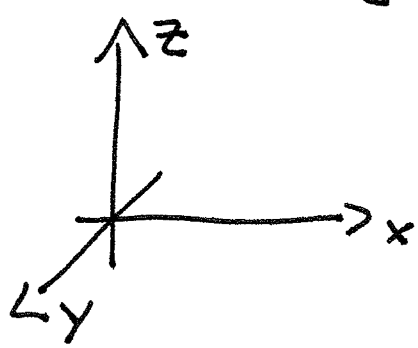
Visually, either



or



<u>not</u>



Must satisfy the <u>right-hand rule</u>

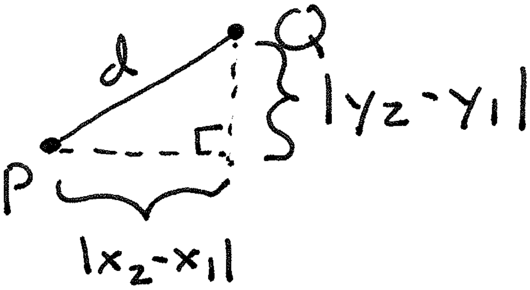
This means: align
- thumb along x-axis
- index finger along y-axis

Then
- z-axis coming out of palm.

How to measure distance?

In $\mathbb{R}^2$, distance from $P=(x_1, y_1)$
to $Q=(x_2, y_2)$

is given by the Pythagorean theorem:



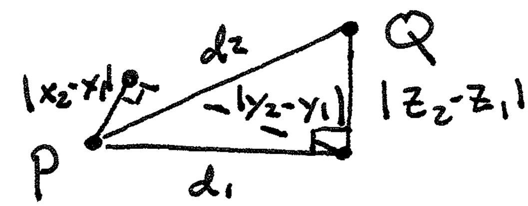
$$d = \sqrt{(x_2-x_1)^2 + (y_2-y_1)^2}$$

Same idea in $\mathbb{R}^3$ (or in $\mathbb{R}^n$ for any n)

The distance from $P=(x_1, y_1, z_1)$
to $Q=(x_2, y_2, z_2)$

is given by

$$d_2 = \sqrt{d_1^2 + (z_2-z_1)^2}$$



$$= \sqrt{(x_2-x_1)^2 + (y_2-y_1)^2 + (z_2-z_1)^2}$$

This allows us to use algebra to describe a geometric object: the sphere of radius r centered at (a, b, c) is the collection of all (x, y, z) such that the distance from (x, y, z) to (a, b, c) is r.

So (x, y, z) must satisfy

$$\sqrt{(x-a)^2 + (y-b)^2 + (z-c)^2} = r,$$

or $$(x-a)^2 + (y-b)^2 + (z-c)^2 = r^2.$$

The unit sphere in $\mathbb{R}^3$ is the sphere of radius 1 centered at the origin.

$$x^2 + y^2 + z^2 = 1$$