Lecture 25

Oct. 24 2011

Exam 2 info

Average ~ 31

Letter Grade	Score	# Students
A	≥ 35	~80
B	≥ 29	~80
C	≥ 22	~70
D	≥ 16	~ 10
F	< 16	~ 15

Last time: Center of mass is $(\bar{x}, \bar{y})$ where

$$\bar{x} = \iint_R x\rho(x,y)\,dA, \quad \bar{y} = \iint_R y\rho(x,y)\,dA \quad (\rho \text{ density function})$$

This week: Triple integrals & Change of coordinates.

Goal: define $\iiint_R f(x,y,z)\,dV$

Start with simple case: $R = [a,b] \times [c,d] \times [p,q]$

Same idea as before: subdivide box into sub-boxes, take sample points, and add.

$$\text{Then } \iiint_R f\,dV = \lim_{l,m,n \to \infty} \underbrace{\sum_{i=1}^{l}\sum_{j=1}^{m}\sum_{k=1}^{n} f(x_i^*, y_j^*, z_k^*)\,\Delta x\,\Delta y\,\Delta z}$$

dV = "volume form"

$\boxed{\text{Fubini}}$

$$= \int_a^b \int_c^d \int_p^q f(x,y,z)\,dz\,dy\,dx$$

$$= \int_p^q \int_c^d \int_a^b f(x,y,z)\,dx\,dy\,dz$$

$\boxed{\text{6 possible orderings!!!}}$

More general regions R:

"type I" $\rightsquigarrow$ $R = \{(x,y,z) \mid g_1(x,y) \le z \le g_2(x,y)\}$

(dumb convention) (x,y) in region D in (x,y)-plane

e.g. The sphere of radius r

$g_1(x,y) = -\sqrt{r^2-x^2-y^2}$, $g_2(x,y) = \sqrt{r^2-x^2-y^2}$,

D = disc of radius r

Then above can be interpreted as

$$\iiint_R f \, dV = \iint_D \underbrace{\left[\int_{g_1(x,y)}^{g_2(x,y)} f(x,y,z)\,dz\right]}_{A(x,y)} dA$$

double integral

$\iint_D A(x,y)\,dA$ handled as before

e.g. R = sphere of radius r, $f(x,y,z) = 1$.

$$\text{Volume} = \iiint_R dV = \iint_D \left(\int_{-\sqrt{r^2-x^2-y^2}}^{\sqrt{r^2-x^2-y^2}} dz\right) dA$$

$$= \int_{-r}^{r} \int_{-\sqrt{r^2-x^2}}^{\sqrt{r^2-x^2}} \int_{-\sqrt{r^2-x^2-y^2}}^{\sqrt{r^2-x^2-y^2}} dz\,dy\,dx$$

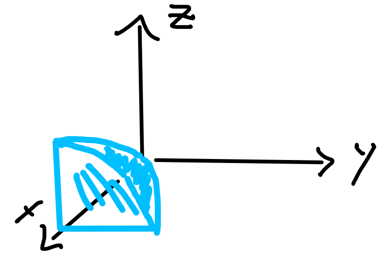
$$= \int_{-r}^{r} \left(\text{Area disc radius } \sqrt{r^2-x^2}\right) dx$$

$$= \int_{-r}^{r} \pi(r^2-x^2)\,dx = 2\pi r^3 - 2\pi \frac{r^3}{3} = \frac{4}{3}\pi r^3$$

Alternatives: bound y between $g_1(x,z) \le y \le g_2(x,z)$

or bound x $g_1(y,z) \le x \le g_2(y,z)$.

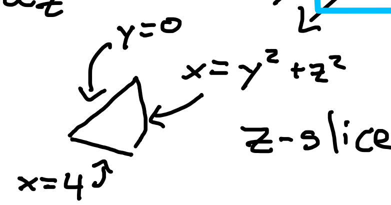
Ex Find volume of solid R in 1^{st} octant bounded by $x=4$, $x=y^2+z^2$, $y=0$, $z=0$



Several methods:

1) $$\int_0^2 \int_0^{\sqrt{4-z^2}} \int_{y^2+z^2}^4 dx\,dy\,dz$$

slice first in z



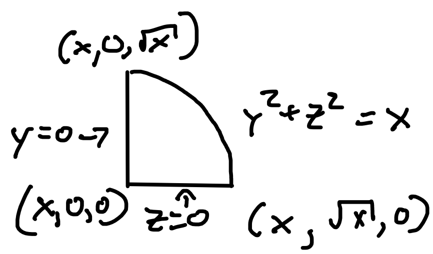
or

2) $$\int_0^2 \int_{z^2}^4 \int_0^{\sqrt{z^2-x}} dy\,dx\,dz$$

or

3) $$\int_0^4 \int_0^{\sqrt{x}} \int_0^{\sqrt{x-y^2}} dz\,dy\,dx$$

slice first in x



or

4) $$\int_0^2 \int_0^{\sqrt{4-y^2}} \int_{y^2+z^2}^4 dx\,dz\,dy$$

slice first in y.

Note: In approach 3), recognize $\int_0^{\sqrt{x}} \int_0^{\sqrt{x-y^2}} dz\,dy = \frac{1}{4}$ area of disc of radius $\sqrt{x}$

So get $V = \int_0^4 \frac{1}{4}\pi x\,dx = \frac{4^2}{8}\pi = 2\pi$.