Lecture 6

Sept. 2 2011

Last time: graphing functions of several variables

The graph of $f(x,y)$ is a surface in $\mathbb{R}^3$.

Today: Quadric surfaces (§ 12.6)

Start with review of conic sections (in $\mathbb{R}^2$)

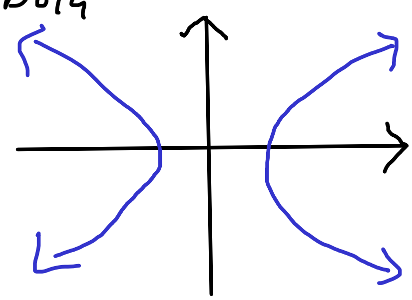
Solution set to equation

$$Ax^2 + Bxy + Cy^2 + Dx + Ey + F = 0$$

can have several forms, depending on the discriminant $B^2 - 4AC$.

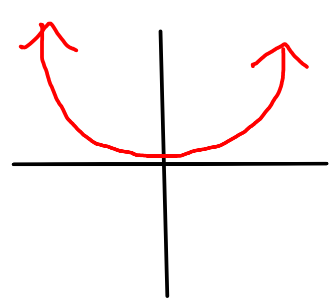
Case I: disc. > 0 hyperbola

Example $x^2 - y^2 - 1 = 0$



Case II: disc. $= 0$ parabola

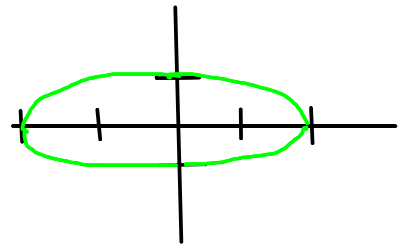
Example: $x^2 - y = 0$



Case III: disc. < 0 ellipse or circle

Example

$$x^2 + 4y^2 - 4 = 0$$



Jump up one dimension:

Quadric surfaces are the solution sets to equations of the form

$$Ax^2 + By^2 + Cz^2 + Dxy + Eyz + Fxz + Gx + Hy + Iz + J = 0$$

Actually, this can usually be reduced to either

$$Ax^2 + By^2 + Cz^2 + J = 0$$

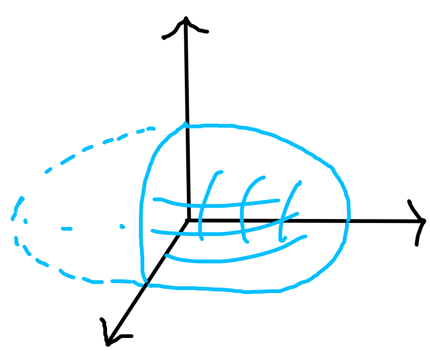
or

$$Ax^2 + By^2 + Iz = 0$$

Important cases:

$\boxed{\text{ellipsoid}}$:

$$\frac{x^2}{A^2} + \frac{y^2}{B^2} + \frac{z^2}{C^2} = 1$$

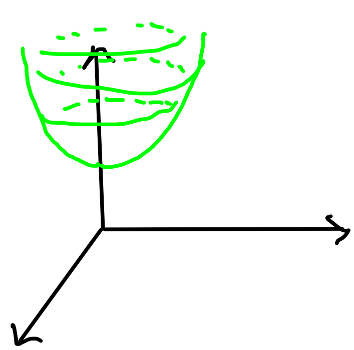


Cross-sections in x, y, and z directions all ellipses.

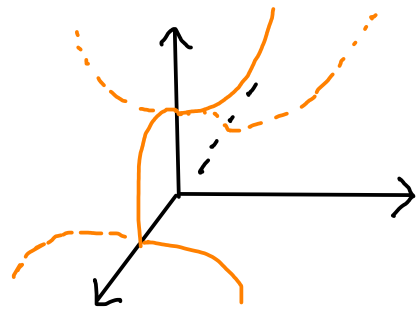
$\boxed{\text{(elliptic) paraboloid}}$:

$$z = Ax^2 + By^2$$

same sign



Cross-sections in z-direction are ellipses
Cross-sections in x or y directions are parabolas.



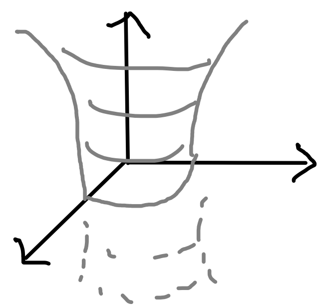
$\boxed{\text{(hyperbolic) paraboloid}}$:

$$z = Ax^2 + By^2$$

(saddle) $\quad$ opposite signs

Cross-sections in z-direction are hyperbolas
Cross-sections in x or y directions are parabolas.

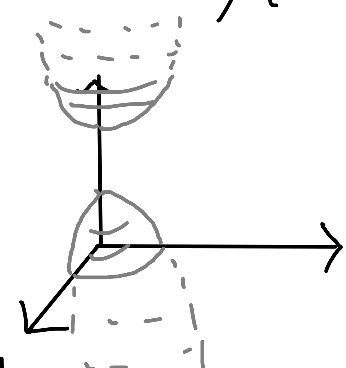
$\boxed{\text{hyperboloid}}$:



$$\frac{x^2}{A^2} + \frac{y^2}{B^2} - \frac{z^2}{C^2} = 1$$

Cross-sections in z-direction are ellipses.
Cross-sections in x or y directions are hyperbolas

Also "two sheeted" version



$$-\frac{x^2}{A^2} - \frac{y^2}{B^2} + \frac{z^2}{C^2} = 1$$

Same cross-section types

$\boxed{\text{See link on course webpage.}}$

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§14.2 Limits & Continuity (for functions of several variables)

Recall For $f: \mathbb{R} \to \mathbb{R}$, say f is continuous at c if

$$\boxed{f(c) = \lim_{x \to c} f(x)}$$

Also makes perfect sense for $f: \mathbb{R}^2 \to \mathbb{R}$,

if we can make sense of $\lim_{(x,y)\to(a,b)} f(x,y)$.

For $f: \mathbb{R} \to \mathbb{R}$, what do we mean by $\lim_{x \to c} f(x) = L$?

Rough definition: if x is very close to c, but not equal to c, then $f(x)$ is very close to L.

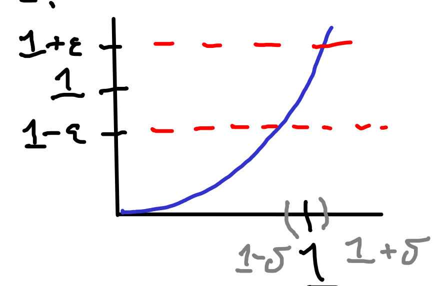
Alternatively: in order to guarantee that $f(x)$ is within error ε of the value L, we need to pick x within distance δ from the point c.

Example: To cut square sheet of area within given tolerance of $1\,m^2$, how close to $1\,m$ must sides be?

Precise definition: $\lim_{x\to c} f(x) = L$ if for every $\varepsilon > 0$ there is a $\delta > 0$ so that whenever $0 < |x - c| < \delta$ then $|f(x) - \varepsilon|$.

Back to example. $f(x) = x^2$. Want

$$1 - \varepsilon < x^2 < 1 + \varepsilon.$$



$f(x)$ steeper to the right of 1.

Actual error will be $\leq 2\delta + \delta^2$, so want to force $2\delta + \delta^2 < \varepsilon$

If we pick $\delta < \varepsilon/4$ and also < 2, then

$$2\delta + \delta^2 < 2\delta + 2\delta = 4\delta < 4(\varepsilon/4) = \varepsilon.$$