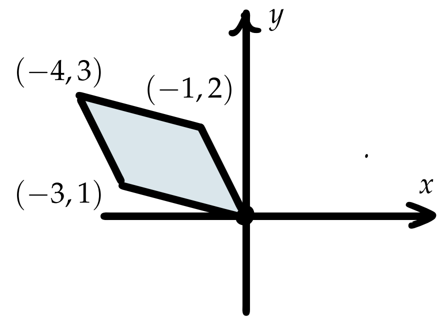
Math 241 - Solutions to Quiz 5- Thursday, November 10

1. Let P be the parallelogram in the plane in the figure to the right. Write C for the boundary curve of P.



   (a) Let $F(x,y) = (-y^2 + y, x^2 + y)$. Use Green's theorem to rewrite $\int_C F \cdot d\mathbf{r}$ as a double integral. (2 points)
   SOLUTION:
   We take C with counterclockwise orientation. Applying Green's theorem gives

$$\int_C F \cdot d\mathbf{r} = \int\int_P \frac{\partial Q}{\partial x} - \frac{\partial P}{\partial y} dA = \int\int_P 2x + 2y - 1 dA$$

   (b) Use a change of coordinates to rewrite your answer from part (a) as a double integral over the unit square $[0,1] \times [0,1]$. (3 points)
   SOLUTION:
   Use the transformation $T(u,v) = (-u - 3v, 2u + v)$. T takes the unit square to P. Then $J(T) = 5$ and we have

$$\int\int_P 2x + 2y - 1 dA = \int_0^1 \int_0^1 2(-u - 3v) + 2(2u + v) - 1(5dudv)$$

$$= 5\int_0^1 \int_0^1 2u - 4v - 1dudv$$

   (c) Evaluate the integral you found in part (c). (2 points)
   SOLUTION:
   -10

2. Let S be the surface parametrized by $\mathbf{r}(u,v) = (u^2 + 1, v^3 + 1, u + v)$. Find an equation for the tangent plane to S at the point $\mathbf{r}(1,1)$. (3 points)

   SOLUTION:

   $\mathbf{r}_u = (2u, 0, 1)$ and $\mathbf{r}_v = (0, 3v^2, 1)$. We have

$$\mathbf{r}_u \times \mathbf{r}_v = \det \begin{pmatrix} i & j & k \\ 2u & 0 & 1 \\ 0 & 3v^2 & 1 \end{pmatrix} = (-3v^2, -2u, 6uv^2)$$

   So a normal to the surface at $\mathbf{r}(1,1) = (2,2,2)$ is given by $(-3,-2,6)$ and an equation for the tangent plane is $-3(x-2) - 2(y-2) + 6(z-2) = 0$.