Solutions to Quiz 3 - Thursday, October 13

1. Find the arc length of the curve C described by $x = y^{3/2}$ and with endpoints $(0,0)$ and $(\frac{8}{3\sqrt{3}}, \frac{4}{3})$. (4 points)

   SOLUTION:

   Arc Length=

$$\int_{y_0}^{y_1} \sqrt{1 + (\frac{dx}{dy})^2} dy = \int_0^{4/3} \sqrt{1 + (3/2y^{1/2})^2} dy = \int_0^{4/3} \sqrt{1 + \frac{9y}{4}} dy = 56/27$$

2. Let $$\mathbf{F}(x,y) = (y^2 \cos x, 2y \sin x + 1)$$
   and let C be the curve from $(0,0)$ to $(\frac{\pi}{2}, \frac{\pi^3}{8})$ parametrized by $\mathbf{r}(t) = (t, t^3)$. Use the Fundamental Theorem for Line Integrals to compute $\int_C \mathbf{F} \cdot d\mathbf{r}$. (4 points)

   SOLUTION:

   Note that $\mathbf{F}(x,y) = \nabla(f)$, where $f = y^2 \sin x + y$. By the Fundamental Theorem for Line Integrals,

$$\int_C \mathbf{F} \cdot d\mathbf{r} = f(\frac{\pi}{2}, \frac{\pi^3}{8}) - f(0,0) = (\pi^3/8)^2 + \pi^3/8$$

3. You are asked to paint one side of a fence. The fence is not straight; the curve traced out by the base is a quarter circle with radius 2. For convenience, let us imagine this is the quarter circle in the first quadrant (with endpoints $(2,0)$ and $(0,2)$). The height of the fence varies according to the x coordinate by the function $6 - \frac{x}{10}$.

   Set up, but **do not solve**, an integral to measure the area of the fence to be painted. (2 points)

   SOLUTION:

   Set $h(x,y) = 6 - \frac{x}{10}$ and C to be the quarter circle in the first quadrant with radius 2 parametrized by $x = 2\cos t$, $y = 2\sin t$, $0 \leq t \leq \pi/2$. Then the surface area is given by the integral $\int_C h ds$. We have $ds = \sqrt{(dx/dt)^2 + (dy/dt)^2} = 2$. So

$$\int_C h ds = \int_0^{\pi/2} (6 - \frac{2\cos t}{10}) 2 dt = \int_0^{\pi/2} 12 - \frac{2\cos t}{5} dt.$$