Worksheet 11: 10/25/11

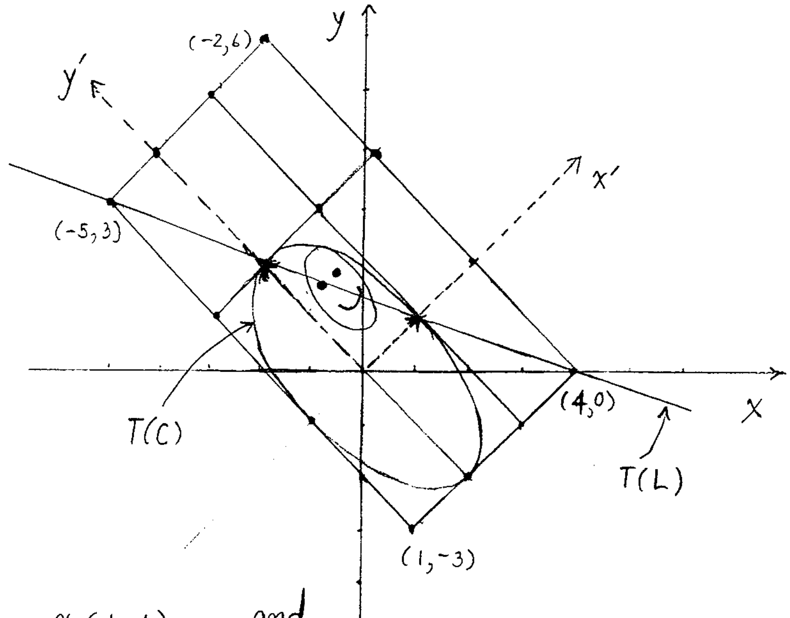
1. (a)

$$T(-1,2) = (-1-2(2), -1+2(2)) = (-5,3)$$

$$T(-1,-1) = (-1-2(-1), -1+2(-1)) = (1,-3)$$

$$T(2,2) = (2-2(2), 2+2(2)) = (-2,6)$$

$$T(2,-1) = (2-2(-1), 2+2(-1)) = (4,0)$$

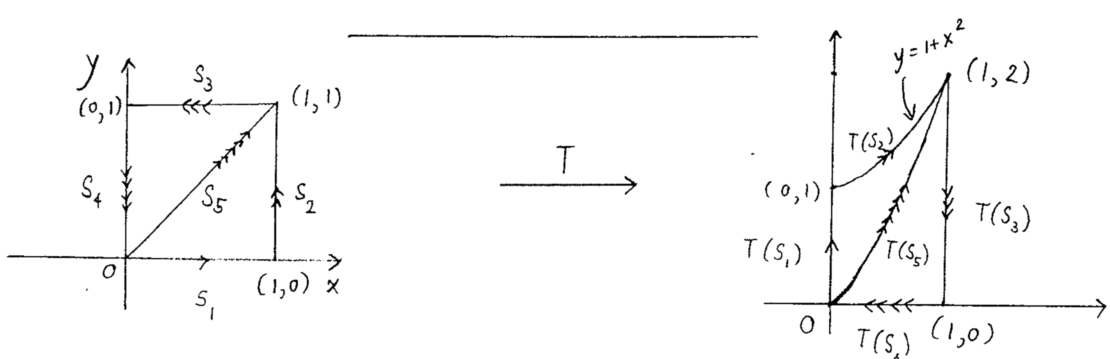


1. (c) On x-axis: $T(x,0) = (x,x) = x(1,1)$, and
on y-axis: $T(0,y) = (-2y, 2y) = y(-2,2)$, shown as x', y' in the picture.

1. (d) Parametrize L: $r(t) = (t, 1-t)$, $t \in \mathbb{R}$.
Then $f(t) = T(r(t)) = (t-2(1-t), t+2(1-t)) = (3t-2, 2-t)$, $t \in \mathbb{R}$, a line.

1. (e) C: $r(t) = (\cos t, \sin t)$, $0 \le t \le 2\pi$
$T(C) = (\cos t - 2\sin t, \cos t + 2\sin t)$, $0 \le t \le 2\pi$, an ellipse.

2.



S_1: $r_1(t) = (t,0)$, $t \in [0,1]$
$T(r_1(t)) = T(t,0) = (0,t)$, $t \in [0,1]$, line segment $x=0$, $0 \le y \le 1$.

S_2: $r_2(t) = (1,t)$, $t \in [0,1]$
$T(r_2(t)) = T(1,t) = (t, 1+t^2)$, $t \in [0,1]$, a part of the parabola $y = 1+x^2$, $0 \le x \le 1$

S_3: $r_3(t) = (t, 1)$, $t \in [0,1]$.

$T(r_3(t)) = T(t,1) = (1, 2t)$, $t \in [0,1]$, the line segment $x=1, 0 \leq y \leq 2$.

S_4: $r_4(t) = (0,t)$, $t \in [0,1]$.

$T(r_4(t)) = T(0,t) = (t,0)$, $t \in [0,1]$, the line segment $y=0$, $0 \leq x \leq 1$.

S_5: $r_5(t) = (t,t)$, $t \in [0,1]$.

$T(r_5(t)) = T(t,t) = (t, t+t^3)$, $t \in [0,1]$, part of $y = x + x^3$, $0 \leq x \leq 1$.

---

3. (a) Since rotation preserves length, we have $T(r,\theta) = (r, \theta + \frac{\pi}{4})$.

3. (b) Write $x = r\cos\theta$ and $y = r\sin\theta$.

Applying part (a), we get

$$T(x,y) = \left(r\cos\left(\theta + \frac{\pi}{4}\right), \ r\sin\left(\theta + \frac{\pi}{4}\right)\right)$$

Consider
$$\begin{aligned} r\cos\left(\theta + \frac{\pi}{4}\right) &= r\cos\theta\cos\frac{\pi}{4} - r\sin\theta\sin\frac{\pi}{4} \\ &= x\cos\frac{\pi}{4} - y\sin\frac{\pi}{4} \\ &= \frac{\sqrt{2}}{2}x - \frac{\sqrt{2}}{2}y \end{aligned}$$

and
$$\begin{aligned} r\sin\left(\theta + \frac{\pi}{4}\right) &= r\sin\theta\cos\frac{\pi}{4} + r\cos\theta\sin\frac{\pi}{4} \\ &= y\cos\frac{\pi}{4} + x\sin\frac{\pi}{4} \\ &= \frac{\sqrt{2}}{2}x + \frac{\sqrt{2}}{2}y \end{aligned}$$

So we get $T(x,y) = \left(\frac{\sqrt{2}}{2}x - \frac{\sqrt{2}}{2}y, \ \frac{\sqrt{2}}{2}x + \frac{\sqrt{2}}{2}y\right)$.
