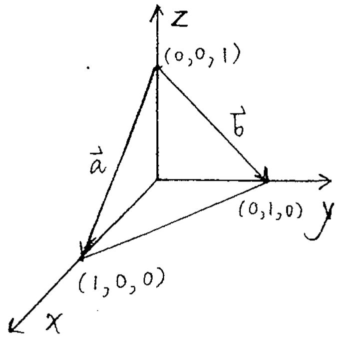
Worksheet 13 Solutions : 11/8/11

1. (a)



1. (b) $\vec{r}(u,v) = (u, v, 1-u-v)$, where

$$D = \{ (u,v) \mid 0 \leq u \leq 1 \ , 0 \leq v \leq 1-u \}$$

1. (c) $\text{Area}(S) = \iint_D |\vec{r}_u \times \vec{r}_v| \, du dv$

We have $\vec{r}_u = (1,0,-1)$ and $\vec{r}_v = (0,1,-1)$,

so $|\vec{r}_u \times \vec{r}_v| = |(1,0,-1) \times (0,1,-1)| = |(1,1,1)| = \sqrt{3}$.

Then $\text{Area}(S) = \int_0^1 \int_0^{1-u} \sqrt{3} \, dv \, du$

$$= \sqrt{3} \int_0^1 (1-u)\, du = \sqrt{3}\left[u - \frac{u^2}{2} \right]_0^1 = \frac{\sqrt{3}}{2}.$$

1. (d) From picture in 1. (a),

$$\text{Area}(S) = \frac{1}{2}|\vec{a} \times \vec{b}| = \frac{1}{2}\left|(1,0,-1) \times (0,1,-1)\right| = \frac{1}{2}|(1,1,1)| = \frac{\sqrt{3}}{2}.$$

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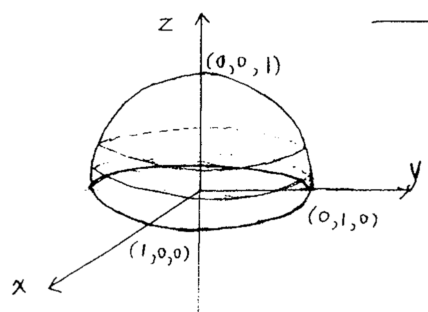
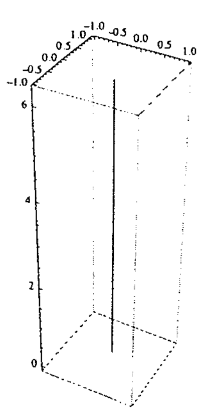
2. (a)



2. (b) $\vec{r}(r,\theta) = (r\cos\theta, r\sin\theta, 1-r^2)$,

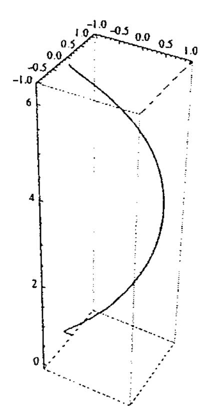
where $0 \leq r \leq 1$; $0 \leq \theta \leq 2\pi$.

3. (a)

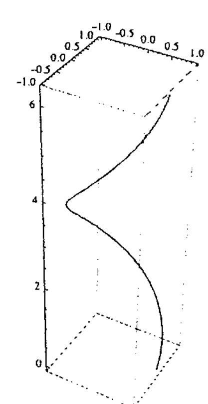


when $u = 0$

3. (b)

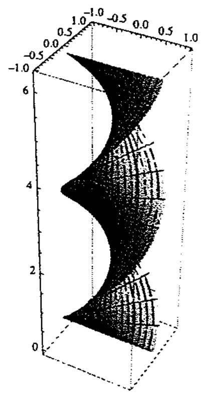


when $u = -1$

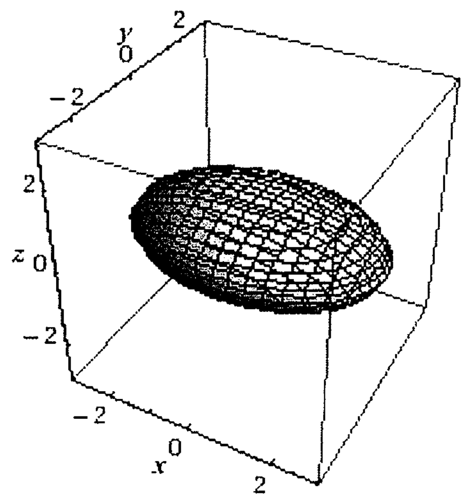


when $u = 1$

3. (c)



4. (a)



4. (b) Let $T(x,y,z) = (3x, 2y, z)$.

Then T takes the unit sphere S to E.

We have the parametrization of the sphere S given by

$x = \sin\phi\cos\theta$, $y = \sin\phi\sin\theta$, $z = \cos\phi$, $\phi \in [0,\pi]$, $\theta \in [0,2\pi]$

So the parametrization for E is

$r(\phi,\theta) = (3\sin\phi\cos\theta, 2\sin\phi\sin\theta, \cos\phi)$,

where $0 \leq \phi \leq \pi$, $0 \leq \theta \leq 2\pi$