Thursday, September 8 $**$ *Functions of several variables.*

1. Consider the function
$$f(x,y) = \frac{2xy}{x^2+y^2}.$$
   (a) What does this function look like along a line $y = mx$?

   (b) Sketch the graph of $f(x,y)$.

2. Consider the function
$$f(x,y) = xy.$$
   (a) Sketch the level sets of f.

   (b) Sketch the graph of $f(x,y)$. What is the name of this surface ?

3. Let $f(x,y) = 3x+5y-1$. This problems deals with
$$\lim_{(x,y)\to(1,1)} 3x+5y-1.$$
   (a) Let $\varepsilon = 1$. Find a $\delta > 0$ such that if $\|(x,y)-(1,1)\| < \delta$, then $|f(x,y)-7| < \epsilon$.

   (b) Now find a $\delta > 0$ for arbitrary ε (your answer should be in terms of ε).

4. In class, we showed that
$$\lim_{(x,y)\to(1,0)} \frac{x}{y}$$
does not exist, by approaching the point $(1,0)$ along different lines. This can also be shown directly from the ε, δ definition. To do this, for each possible real number L, you must show that the limit cannot be L.

   (a) Let L be any real number. For the value $\varepsilon = 1$, show that no matter which $\delta > 0$ is chosen, there is always a point (x,y) such that $\|(x,y)-(1,0)\| < \delta$ but $|\frac{x}{y} - L| \geq 1$. This shows that the limit is not L.

   (**Hint:** Take *any* value for x in the interval $(1-\delta, 1+\delta)$. Show that there is a value for y that makes the above inequalities true.)

   (b) More generally, show that for *any* $\varepsilon > 0$, no good δ can be found.