Tuesday, September 13 ∗∗ *Partial derivatives.*

1. Let $f(x,y) = y^2 \ln(x^3+1) + \sqrt{y}$. Find the partial derivatives

$$f_x, \qquad D_2 f, \qquad \frac{\partial^2 f}{\partial x^2}, \qquad D_1 D_2 f, \qquad f_{yy}, \qquad \frac{\partial^2 f}{\partial y \partial x}.$$

2. (Harmonic functions) The partial differential equation

$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2} = 0$$

is called the **Laplace equation**. Any function $u(x,y,z)$ satisfying the Laplace equation is called a **harmonic function**.

   (a) Let $u(x,y) = e^{ax}\cos(bx)$. Find u_{xx} and u_{yy}. What must be true of a and b in order for u to be harmonic of two variables?

   (b) Show that $u(x,y,z) = \frac{1}{\sqrt{x^2+y^2+z^2}}$ is a harmonic function of three variables.

3. (Counterexample to Clairaut) Let

$$f(x) = \begin{cases} xy\frac{x^2-y^2}{x^2+y^2}, & (x,y) \neq (0,0) \\ 0, & (x,y) = (0,0). \end{cases}$$

   (a) Find $f_x(0,y)$.

   (b) Find $f_y(x,0)$.

   (c) Show that $f_{xy}(0,0) \neq f_{yx}(0,0)$.

4. The wind-chill index $W = f(T,v)$ is the perceived temperature when the actual temperature is T and the wind speed is v. Here is a table of values for W.

Wind speed (km/h)

Actual temperature (°C)

T \ v	20	30	40	50	60	70
−10	−18	−20	−21	−22	−23	−23
−15	−24	−26	−27	−29	−30	−30
−20	−30	−33	−34	−35	−36	−37
−25	−37	−39	−41	−42	−43	−44

   (a) Use the table to estimate $\frac{\partial f}{\partial T}$ and $\frac{\partial f}{\partial v}$ at $(T,v) = (-20,40)$.

   (b) Use your answer in (a) to write down the linear approximation to f at $(-20,40)$.

   (c) Use your answer in (b) to approximate $f(-22,45)$.