Tuesday, October 11 ** *Integrating vector fields*

1. Consider the function $f(x, y) = 8y$. Let C be the curve $x = y^2 - 1$ between the points $P = (-1, 0)$ and $Q = (0, 1)$.

   (a) Find a parametrization $\mathbf{r}(t)$ for C starting at P and ending at Q.

   (b) Using the parametrization from (a), compute $\int_C f\, ds$.

   (c) Find a parametrization $\mathbf{q}(t)$ for C starting at Q and ending at P, and use this to calculate $\int_C f\, ds$. Did you get the same answer as in (b)?

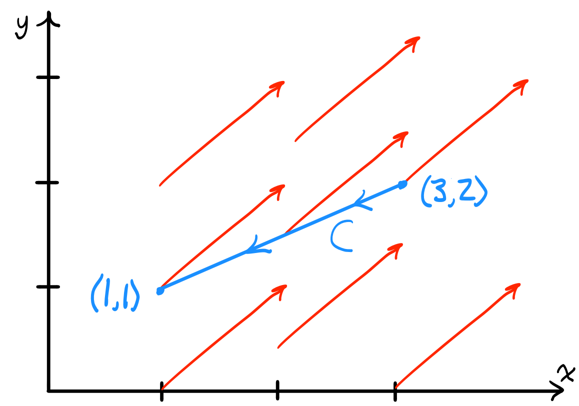
   (d) Using the two parametrizations from (a) and (c), calculate $\int_C f\, dy$. Do you get the same answer in both cases?

   (e) Write $\mathbf{F} = \nabla(f)$. Compute $\int_C \mathbf{F} \cdot d\mathbf{r}$ and $\int_C \mathbf{F} \cdot d\mathbf{q}$. Do you get the same answer in both cases?

2. Consider the curve C and vector field $\mathbf{F}$ shown to the right.

   (a) Without parameterizing C, evaluate $\int_C \mathbf{F} \cdot d\mathbf{r}$. (Hint: Use the Fundamental Theorem for Line Integrals.)

   (b) Find a parameterization of C and use it to check your answer in (a) by computing $\int_C \mathbf{F} \cdot d\mathbf{r}$ explicitly.



3. Consider the vector field $\mathbf{F} = (-y, x)$.

   (a) Let C_1 be the straight line segment from $(1, 0)$ to $(-1, 0)$. Compute $\int_{C_1} \mathbf{F} \cdot d\mathbf{r}$.

   (b) Let C_2 be the upper semicircle (with counter-clockwise orientation). Compute $\int_{C_2} \mathbf{F} \cdot d\mathbf{r}$.

4. Suppose the curve C is contained within a level set of the function f.

   (a) From the point of view of the Fundamental Theoerm, why is $\int_C \nabla(f) \cdot d\mathbf{r} = 0$?

   (b) Give a geometric explanation, not using the Fundamental Theorem, for why this line integral should be zero.