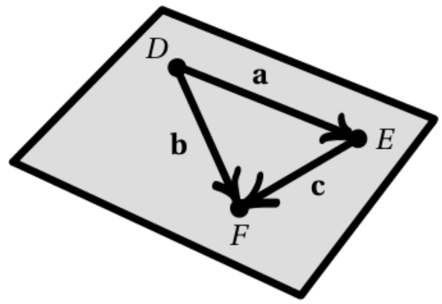
1. (a) Consider the points $A = (0,1,2)$, $B = (2,2,3)$, and $C = (-1,3,4)$. Compute the vectors $\mathbf{v} = \overrightarrow{AB}$ and $\mathbf{w} = \overrightarrow{AC}$. **(2 points)**

$$\vec{v} = B - A = (2,2,3) - (0,1,2) = (2,1,1)$$
$$\vec{w} = C - A = (-1,3,4) - (0,1,2) = (-1,2,2)$$

$\mathbf{v} = \langle 2, 1, 1 \rangle$ $\mathbf{w} = \langle -1, 2, 2 \rangle$

(b) Consider the points $D = (0,2,1)$, $E = (1,4,1)$, and $F = (3,8,2)$ and the vectors $\mathbf{a} = \langle 1,2,0\rangle$, $\mathbf{b} = \langle 3,6,1\rangle$, and $\mathbf{c} = \langle 2,4,1\rangle$ as shown at right. Find a normal vector $\mathbf{n}$ to the plane containing the points D, E, and F. **(3 points)**



$$\vec{n} = \vec{a} \times \vec{b} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 1 & 2 & 0 \\ 3 & 6 & 1 \end{vmatrix}$$
$$= (2\cdot 1 - 0\cdot 6)\vec{i} - (1\cdot 1 - 0\cdot 3)\vec{j} + (1\cdot 6 - 2\cdot 3)\vec{k}$$
$$= 2\vec{i} - \vec{j} = (2,-1,0)$$

Also acceptable:

$\vec{n} = \vec{a} \times \vec{c} = (2,-1,0)$

or $\vec{n} = \vec{b} \times \vec{c} = (2,-1,0)$

or any scalar multiple.

$\mathbf{n} = \langle 2, -1, 0 \rangle$

(c) Let $P = (2,-1,1)$ and $\mathbf{u} = \langle 3,2,4\rangle$. Find a linear equation for the plane that contains P and has normal vector $\mathbf{u}$. **(2 points)**

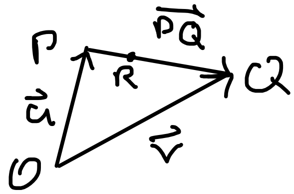
$$3(x-2) + 2(y-(-1)) + 4(z-1) = 0$$
$$3x - 6 + 2y + 2 + 4z - 4 = 0$$
$$3x + 2y + 4z = 8$$

Equation: $3\,x + 2\,y + 4\,z = 8$

(d) For two vectors $\mathbf{v}$ and $\mathbf{w}$ in $\mathbb{R}^3$, which of the following does $|\mathbf{v} \times \mathbf{w}|$ measure? Circle your answer. **(1 point)**

- The length of $\mathbf{v} - \mathbf{w}$.
- The area of the parallelogram determined by $\mathbf{v}$ and $\mathbf{w}$.
- The volume of the parallelepiped determined by $\mathbf{v}$, $\mathbf{w}$, and $\mathbf{v} \times \mathbf{w}$.

2. (a) On the straight-line segment between the points $P = (1,3,-2)$ and $Q = (4,-3,4)$, find the point R that lies one third of the way from P to Q. **(2 points)**



$$R = \vec{u} + \tfrac{1}{3}\vec{PQ}$$
$$= (1,3,-2) + \tfrac{1}{3}(4-1, -3-3, 4-(-2))$$
$$= (1,3,-2) + \tfrac{1}{3}(3,-6,6)$$
$$= (1,3,-2) + (1,-2,2)$$

$\boxed{R = (\ 2\ ,\ 1\ ,\ 0\ )}$

(b) Let L be the line parametrized by $\mathbf{r}(t) = \mathbf{a} + t\mathbf{b}$ for $\mathbf{a} = \langle 1,0,2\rangle$ and $\mathbf{b} = \langle 2,-1,1\rangle$. Find the point Q of intersection of the line L with the plane whose equation is $3x - 2y + z = 14$. **(3 points)**

$$x = 1 + 2t \qquad y = 0 + (-1)t = -t \qquad z = 2 + t$$

$$3(1+2t) - 2(-t) + 2 + t = 14$$
$$3 + 6t + 2t + 2 + t = 14$$
$$9t + 5 = 14$$

$\boxed{t = 1}$ $\quad Q = \vec{a} + \vec{b} = (3,-1,3)$

$\boxed{Q = (\ 3\ ,\ -1\ ,\ 3\ )}$

3. Consider the function $f(x,y) = \dfrac{xy - x^2y}{x^2+y^2}$ for $(x,y) \neq (0,0)$. Evaluate $\lim\limits_{(x,y)\to(0,0)} f(x,y)$ or explain why it does not exist. **(5 points)**

Along line $y = 0$ have

$$\lim_{x\to 0} \frac{0-0}{x^2+0} = 0$$

Along line $y = x$ have

$$\lim_{x\to 0} \frac{x^2 - x^3}{x^2+x^2} = \lim_{x\to 0} \frac{1-x}{1+1} = \frac{1}{2}$$

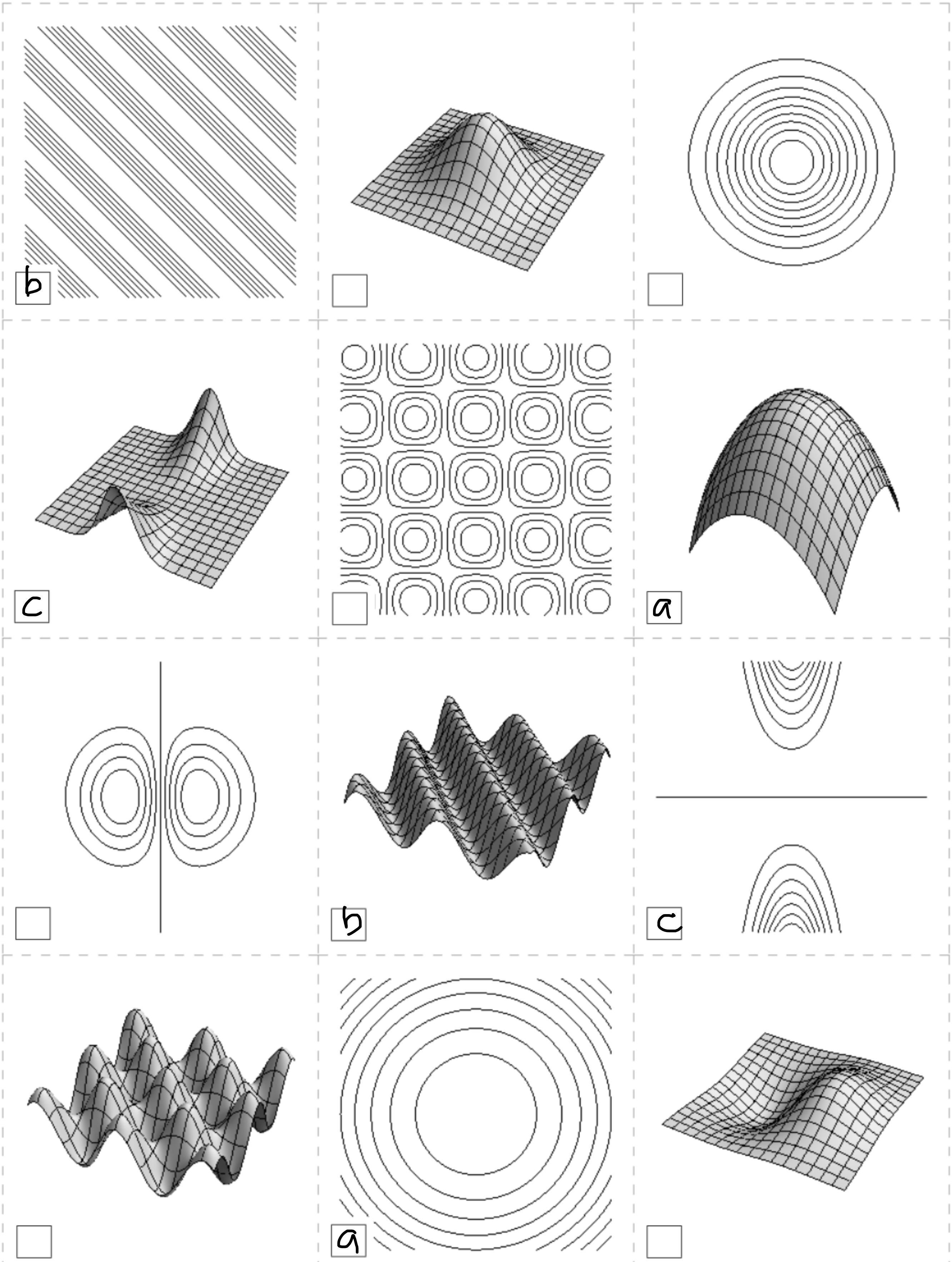
The two limits differ, so

$\lim\limits_{(x,y)\to(0,0)} \dfrac{xy - x^2y}{x^2+y^2}$ Does Not Exist

4. For each function

(a) $-x^2 - y^2$ (b) $\cos(x+y)$ (c) $y^2 e^{-x^2}$

label its graph and its level set diagram from among the options below. Here each level set diagram consists of level sets $\{f(x,y) = c_i\}$ drawn for evenly spaced c_i. **(9 points)**

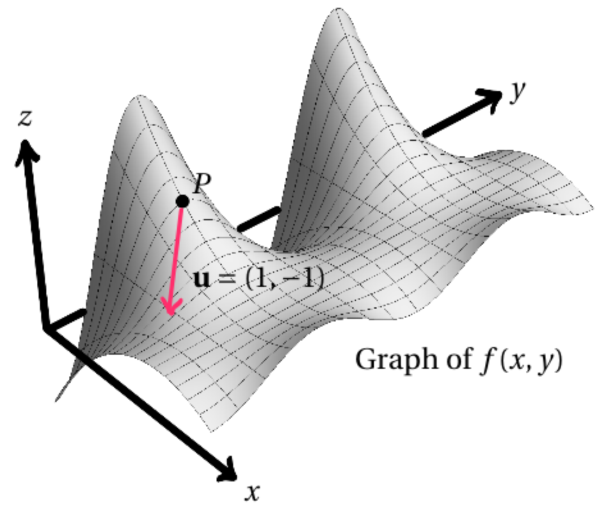


5. (a) Compute the partial derivatives of $f(x,y) = x^3 + 2xy + y$. **(2 points)**

$\dfrac{\partial f}{\partial x} =$ $3x^2 + 2y$

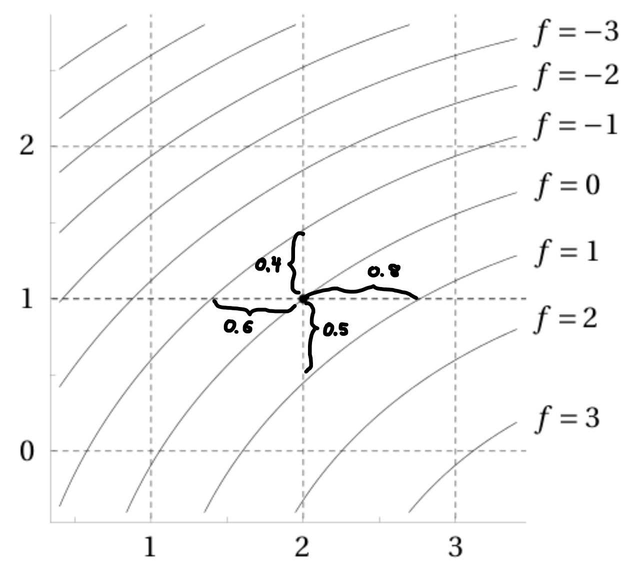
$\dfrac{\partial f}{\partial y} =$ $2x + 1$

(b) Consider the graph of $f\colon \mathbb{R}^2 \to \mathbb{R}$ shown at right. If P is the point $(a, b, f(a,b))$, find the sign of each of the quantities below. Circle your answers. **(1 point each)**



$f_x(a,b)$:	positive	negative	0
$f_y(a,b)$:	positive	negative	0
$D_{\mathbf{u}} f(a,b)$:	positive	negative	0
$f_{xx}(a,b)$:	positive	negative	0

(c) Use the contour plot of $f(x,y)$ shown at right to estimate $f_x(2,1)$ and $f_y(2,1)$. For each, circle the number below that is closest to your estimate. **(4 points)**



$$f_x(2,1) \approx \frac{f(2.8) - f(1.4)}{2.8 - 1.4} = \frac{1 - (-1)}{1.4} \approx 1.5$$

$$f_y(2,1) \approx \frac{f(1.4) - f(0.5)}{1.4 - 0.5} = \frac{-1 - 1}{0.9} \approx -2$$

$f_x(2,1)$:	−3	−2.5	−2	−1.5	−1	−0.5	0	0.5	1	1.5	2	2.5	3
$f_y(2,1)$:	−3	−2.5	−2	−1.5	−1	−0.5	0	0.5	1	1.5	2	2.5	3

6. Suppose a function $f: \mathbb{R}^2 \to \mathbb{R}$ has $f(1,2) = 5$, $\frac{\partial f}{\partial x}(1,2) = -2$, and $\frac{\partial f}{\partial y}(1,2) = 3$. Use linear approximation to estimate $f(1.1, 1.9)$. **(3 points)**

$$f(1.1, 1.9) \approx f(1,2) + \frac{\partial f}{\partial x}(1,2)(1.1-1) + \frac{\partial f}{\partial y}(1,2)(1.9-2)$$
$$= 5 + (-2)\frac{1}{10} + 3\left(-\frac{1}{10}\right)$$
$$= 5 - 0.5 = 4.5$$

$f(1.1, 1.9) \approx$ 4.5

7. Suppose

$$f(x,y): \mathbb{R}^2 \to \mathbb{R}, \quad x(s,t): \mathbb{R} \to \mathbb{R}, \quad \text{and} \quad y(s,t): \mathbb{R} \to \mathbb{R}$$

are functions. Let $F(s,t) = f\big(x(s,t), y(s,t)\big)$ be their composition.

(a) Write the formula for $\frac{\partial F}{\partial s}$ using the Chain Rule. **(1 points)**

$$\frac{\partial F}{\partial s} = \frac{\partial f}{\partial x}\frac{\partial x}{\partial s} + \frac{\partial f}{\partial y}\frac{\partial y}{\partial s}$$

(b) Suppose $x(s,t) = 2s + t$ and $y(s,t) = s^2 t - 1$ and that $f(x,y)$ has the table of values and partial derivatives shown at right. Compute $\frac{\partial F}{\partial s}(2,1)$. **(4 points)**

(x,y)	$f(x,y)$	$\frac{\partial f}{\partial x}$	$\frac{\partial f}{\partial y}$
(2,3)	0	3	6
(2,1)	2	-2	-1
(2,4)	3	4	6
(5,3)	1	3	5

$$\frac{\partial x}{\partial s} = 2 \qquad \frac{\partial y}{\partial s} = 2st$$

$$\frac{\partial F}{\partial s}(2,1) = \frac{\partial f}{\partial x}(x(2,1), y(2,1)) \cdot 2 + \frac{\partial f}{\partial y}(x(2,1), y(2,1)) \cdot 2 \cdot 2 \cdot 1$$
$$= \frac{\partial f}{\partial x}(5,3) \cdot 2 + \frac{\partial f}{\partial y}(5,3) \cdot 4$$
$$= 6 + 20 = 26$$

$\frac{\partial F}{\partial s}(2,1) =$ 26

Extra credit problem: Let $E\colon \mathbb{R}^2 \to \mathbb{R}$ be given by $E(x, y) = 2x + y^2$. Find a $\delta > 0$ so that $|E(\mathbf{h})| < 0.01$ for all $\mathbf{h} = (x, y)$ with $|\mathbf{h}| < \delta$. Carefully justify why the δ you provide is good enough. **(2 points)**

Take $\delta = \frac{1}{1000}$. Then if $\|h\| = \sqrt{x^2+y^2} < \frac{1}{1000}$,

then $|E(x,y)| = |2x+y^2| \leq 2|x| + y^2 < 2 \cdot \frac{1}{1000} + \frac{1}{10^6}$

$< \frac{1}{100}.$

Scratch space below and on back