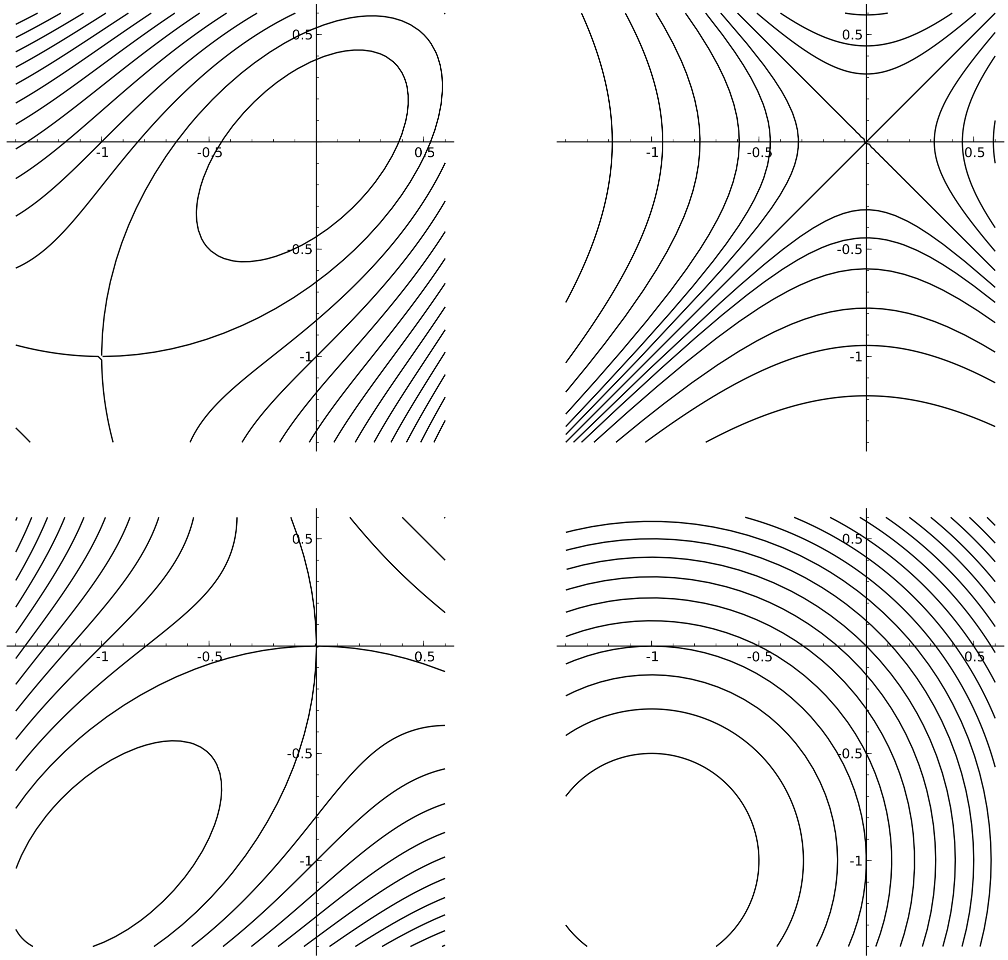
1. Consider the function $f = x^3 + y^3 + 3xy$.

   (a) It turns out the critical points of f are $(0,0)$ and $(-1,-1)$. Classify them into local mins, local maxes, and saddles. **(4 points)**

   (b) Based on your answer in (a), circle the correct contour diagram of f. **(1 point)**



2. Consider the function $f\colon \mathbb{R}^2 \to \mathbb{R}$ given by $f(x, y) = x^2 - 2x + y^2 - 2y$.

   (a) Use Lagrange multipliers to find the max and min of f on the circle $x^2 + y^2 = 8$. **(6 points)**

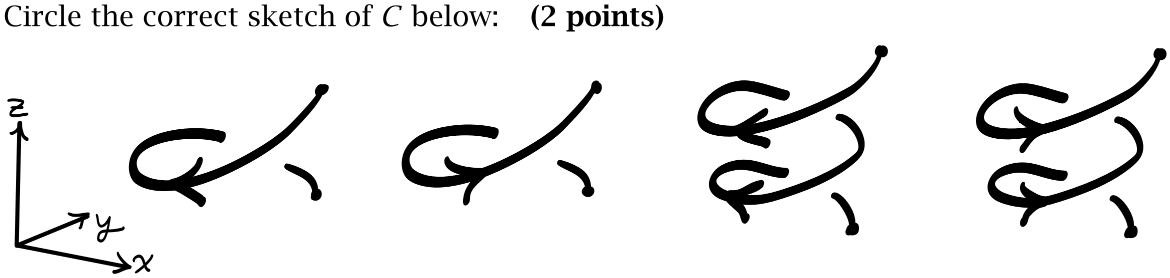
   (b) Consider the region D where $x^2 + y^2 \leq 8$. Explain why f must have a global min and max on D. **(2 points)**

   (c) Find the global min and max of f on D. **(3 points)**

3. Let C be the portion of a helix parameterized by

$$\mathbf{r}(t) = \big(\cos(2t), -\sin(2t), 9 - t\big) \quad \text{for } 0 \leq t \leq 2\pi.$$

   (a) Circle the correct sketch of C below: **(2 points)**

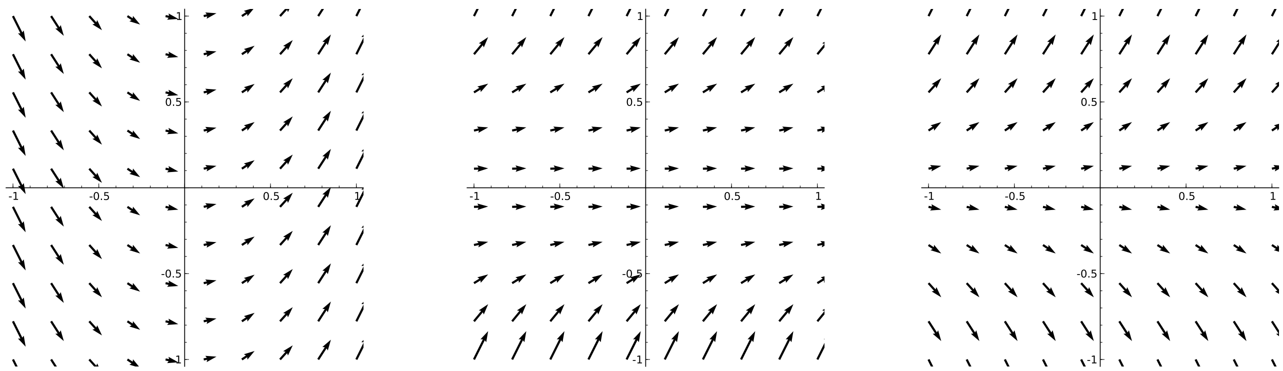


(b) Compute the length of C. **(5 points)**

(c) Suppose C is made of material with density given by $\rho(x, y, z) = x + z$. Give a line integral for the mass of C, and reduce it to an ordinary definite integral (something like $\int_0^1 t^2 \sin t \; dt$). **(3 points)**

4. Let C be the curve parameterized by $\mathbf{r}(t) = (e^t, t)$ for $0 \leq t \leq 1$, and consider the vector field $\mathbf{F} = (1, 2y)$.

(a) Circle the picture of $\mathbf{F}$ below: **(2 points)**



(b) Directly compute $\displaystyle\int_C \mathbf{F} \cdot d\mathbf{r}$. **(5 points)**

(c) The vector field $\mathbf{F}$ is conservative. Find $f\colon \mathbb{R}^2 \to \mathbb{R}$ so that $\nabla f = \mathbf{F}$. **(2 points)**

(d) Use your answer in (c) to check your answer in (b). **(2 points)**

5. Let C be the portion of the ellipse $\dfrac{x^2}{4} + y^2 = 1$ between $A = (0, -1)$ and $B = (0, 1)$ which is shown below left.

(a) Give a parameterization $\mathbf{r}$ of C, indicating the domain so that it traces out precisely the segment indicated. **(3 points)**

(b) Let L be the line segment joining B to A. Give a parameterization $\mathbf{f}\colon [0, 1] \to \mathbb{R}^2$ of L so that $\mathbf{f}(0) = B$ and $\mathbf{f}(1) = A$. **(2 points)**

(c) Suppose $g\colon \mathbb{R}^2 \to \mathbb{R}$ is a function whose level sets are indicated below right. Circle the sign of $\int_C g \, ds$ **(1 point)**

positive   negative   0