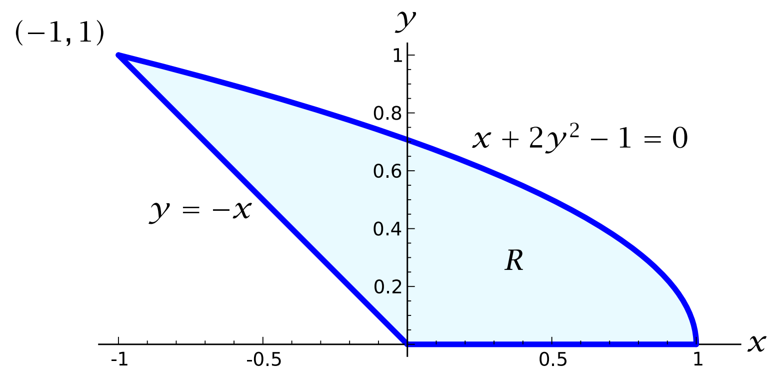
Practice exam for Midterm 3 in Math 241

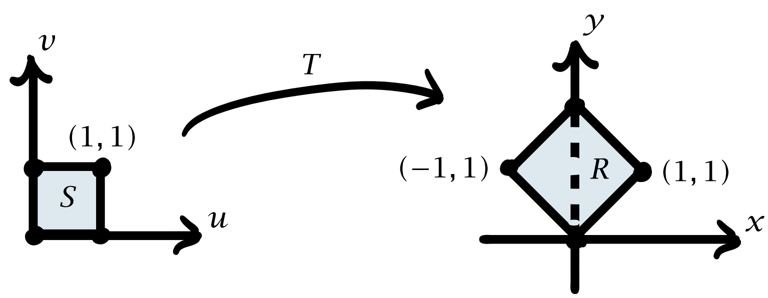
Important note: Several of the problems ask you to "completely setup but not evaluate" a certain integral. This means that all the limits of integration are specified, and the integrand is in terms of the final variables. For example, if S is a surface in $\mathbb{R}^3$, then an acceptable answer for setting up $\iint_S (x+y)^2\, dA$ would be something like $\int_0^2 \int_0^{1-v} (u^2 + v \sin u)\, du\, dv$.

1. Let R be the region shown at right. Evaluate $\iint_R y\, dA$. **(7 points)**



2. Consider the solid region E in the positive octant cut off by $x + y + z = 1$. Completely setup, but do not evaluate, a triple integral which computes the volume of E. **(6 points)**

3. Let R be the region shown at right.

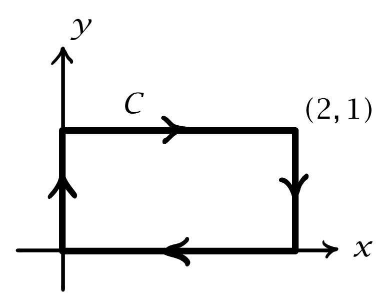


   (a) Find a transformation $T\colon \mathbb{R}^2 \to \mathbb{R}^2$ taking $S = [0,1] \times [0,1]$ to R. **(4 points)**

   (b) Use your change of coordinates to evaluate $\int_R (x+y)^2\, dA$ via an integral over S. **(6 points)**

   Emergency backup transformation: If you can't do (a), pretend you got the answer $T(u,v) = (uv, v)$ and do part (b) anyway.

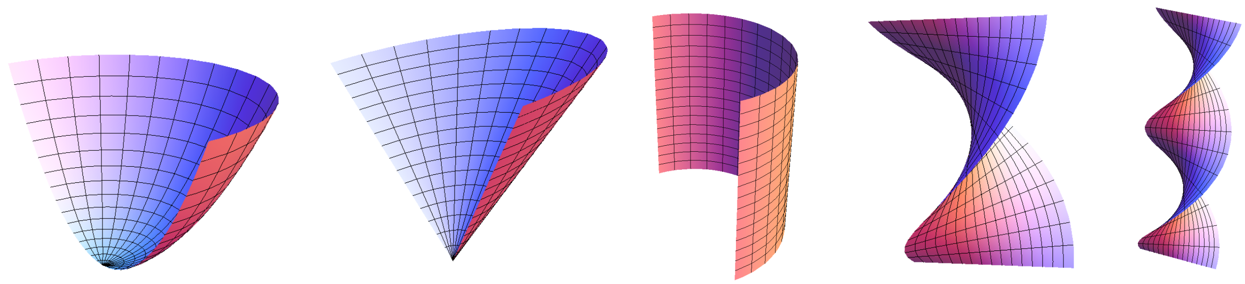
4. Let C be the oriented curve in $\mathbb{R}^2$ shown at right. For the vector field $F(x,y) = (x^3, x^2)$, use Green's theorem to evaluate $\int_C \mathbf{F} \cdot d\mathbf{r}$. **(6 points)**



5. Let E be the portion of the positive octant which is inside the unit sphere. Use spherical coordinates to completely setup, but not evaluate, the integral $\iiint_E x + z\, dV$. **(6 points)**

6. Let S be the surface in $\mathbb{R}^3$ parameterized by $\mathbf{r}(u,v) = (v\cos u, v\sin u, u)$ for $0 \leq u \leq \pi$ and $-1 \leq v \leq 1$.

   (a) Circle the correct picture of S. **(2 points)**

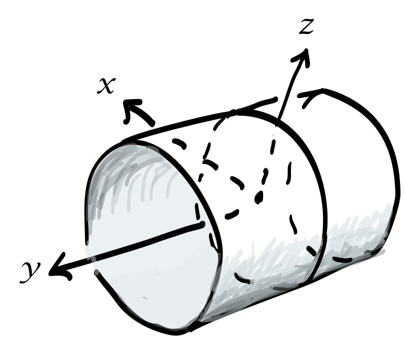


   (b) Completely setup, but do not evaluate, the integral $\iint_S y\, dA$. **(6 points)**

   (c) Find the equation for the tangent plane to S at the point $(0, 0, \pi/2)$ in $\mathbb{R}^3$. **(3 points)**

7. For each surface S below, give a parameterization $\mathbf{r}\colon D \to S$. Be sure to explicitly specify the domain D.

   (a) The portion of the cylinder $x^2 + z^2 = 4$ where $-3 \leq y \leq 3$. **(4 points)**



   (b) The ellipsoid $\dfrac{x^2}{4} + y^2 + \dfrac{z^2}{9} = 1$. **(3 points)**