Math 351 - Elementary Topology
Friday, November 9 $\ast\ast$ *Exam 2 Review Problems*

1. Give an example of subspaces $A \subseteq \mathbb{R}^n$ and $B \subseteq \mathbb{R}^n$, for some n, together with a continuous bijection $f : A \longrightarrow B$ which is *not* a homeomorphism.

2. Show that if $f : X \longrightarrow Y$ is a homeomorphism and $A \subseteq X$, then $\text{Int}\big(f(A)\big) = f(\text{Int}\, A)$.

3. Let $f : X \longrightarrow Y$ be an embedding.

   (a) Prove or disprove: If Y is Hausdorff, so is X.

   (b) Prove or disprove: If X is Hausdorff, so is Y.

4. Show that if $A \subseteq X$ is closed and $B \subseteq Y$ is also closed, then $A \times B \subseteq X \times Y$ is closed. Use **only** the definition of the product topology. In other words, you may *not* use that $\overline{A \times B} = \overline{A} \times \overline{B}$.

5. Let (x_n) and (y_n) be sequences in the spaces X and Y, respectively. Show that $x_n \to x$ and $y_n \to y$ if and only if $(x_n, y_n) \to (x, y)$ in $X \times Y$.

6. Let $X = \mathbb{R}_\ell \times \mathbb{R}$ and let $L \subseteq X$ be a line. Describe the topology on L inherited from X. Hint: the answer depends on the slope of L.

7. Let $X \times Y$ be partitioned into the subsets $X \times \{y\}$, one partition for each $y \in Y$. Show that the resulting quotient $(X \times Y)^*$ is homeomorphic to Y.

8. Give an example of a quotient map $q : X \twoheadrightarrow Y$ such that q is *not* an open map.

9. Let $Z \subseteq \mathbb{R}^2$ be the union of the two coordinate axes. Define $q : \mathbb{R}^2 \twoheadrightarrow Z$ by

$$q(x,y) = \begin{cases} (x,0) & x \neq 0 \\ (0,y) & x = 0. \end{cases}$$

   (a) Show that q is *not* continuous if Z is given the subspace topology.

   (b) Describe the resulting quotient topology on Z. What would be a basis for this topology? Is it Hausdorff?

10. Show that a hexagon with opposite edges glued together with a flip yields $\mathbb{RP}^2$.