

Name: _____

Section: _____

MA 114 QUIZ #7
October 30, 2014

The following quiz is worth 4 points. Each problem will be worth 2 points. Be sure to read the problem carefully and answer every part of the problem. Be sure to show all of your work. **Answers without support will not receive any credit.**

1. Evaluate the following integral: $\int \frac{\sqrt{x^2 - 9}}{x} dx$. The identity $\int \tan^2 \theta d\theta = \tan \theta - \theta$ may be helpful in your computation.

Let $x = 3 \sec \theta$. Then $dx = 3 \sec \theta \tan \theta d\theta$. We have

$$\begin{aligned} \int \frac{\sqrt{x^2 - 9}}{x} dx &= \int \frac{3 \sec \theta \tan \theta \sqrt{(3 \sec \theta)^2 - 9}}{3 \sec \theta} d\theta \\ &= \int 3 \tan^2 \theta d\theta \\ &= \int 3 \sec^2 \theta - 3 d\theta \\ &= 3 \tan \theta - 3\theta + C \\ &= \sqrt{x^2 - 9} - 3 \sec^{-1} \left(\frac{x}{3} \right) + C \end{aligned}$$

Give one point for using the correct substitution and one point for integrating correctly (including writing the final answer in terms of x).

2. Determine the partial fraction decomposition of the following fraction: $\frac{2x + 4}{(x + 1)(x^2 + 1)}$. (Absolutely no integration is involved in this question.)

We have

$$\frac{2x + 4}{(x + 1)(x^2 + 1)} = \frac{C_1}{x + 1} + \frac{C_2 + C_3x}{x^2 + 1}$$

for some constants C_1 , C_2 , and C_3 . Multiplying both sides of the above equation by $(x + 1)(x^2 + 1)$ and equating the coefficients, we have the following system of equations.

$$\begin{aligned} C_1 + C_3 &= 0 \\ C_2 + C_3 &= 2 \\ C_1 + C_2 &= 4 \end{aligned}$$

Therefore, $C_1 = 1$, $C_2 = 3$, and $C_3 = -1$. Thus,

$$\frac{2x + 4}{(x + 1)(x^2 + 1)} = \frac{1}{x + 1} + \frac{3 - x}{x^2 + 1}.$$

Given one point for the correct decomposition and one point for the correct choice of constants.