MA 114 Worksheet # 1: Improper Integrals

1. For each of the following, determine if the integral is proper or improper. If it is improper, explain why. Do ***not*** evaluate any of the integrals.

   (a) $\displaystyle\int_0^2 \frac{x}{x^2-5x+6}\,dx$

   (b) $\displaystyle\int_1^2 \frac{1}{2x-1}\,dx$

   (c) $\displaystyle\int_1^2 \ln(x-1)\,dx$

   (d) $\displaystyle\int_{-\infty}^{\infty} \frac{\sin x}{1+x^2}\,dx$

   (e) $\displaystyle\int_0^{\pi/2} \sec x\,dx$

2. For the integrals below, determine if the integral is convergent or divergent. Evaluate the convergent integrals.

   (a) $\displaystyle\int_{-\infty}^0 \frac{1}{2x-1}\,dx$

   (b) $\displaystyle\int_{-\infty}^{\infty} xe^{-x^2}\,dx$

   (c) $\displaystyle\int_0^2 \frac{x-3}{2x-3}\,dx$
   (write the numerator as $\frac{1}{2}(2x-3)-\frac{3}{2}$)

   (d) $\displaystyle\int_0^{\infty} \sin\theta\,d\theta$

3. Consider the improper integral

$$\int_1^{\infty} \frac{1}{x^p}\,dx.$$

   Integrate using the generic parameter p to prove the integral converges for $p > 1$ and diverges for $p \leq 1$. You will have to distinguish between the cases when $p = 1$ and $p \neq 1$ when you integrate.

4. Use the Comparison Theorem to determine whether the following integrals are convergent or divergent.

   (a) $\displaystyle\int_1^{\infty} \frac{2+e^{-x}}{x}\,dx$

   (b) $\displaystyle\int_1^{\infty} \frac{x+1}{\sqrt{x^6+x}}\,dx$

5. Explain why the following computation is wrong and determine the correct answer. (Try sketching or graphing the integrand to see where the problem lies.)

$$\begin{aligned}\int_2^{10} \frac{1}{2x-8}dx &= \frac{1}{2}\int_{-4}^{12}\frac{1}{u}\,du \\ &= \frac{1}{2}\ln|x|\Big|_{-4}^{12} \\ &= \frac{1}{2}(\ln 12 - \ln 4)\end{aligned}$$

   where we used the substitution

$$\begin{cases} u(x) = 2x-8 \\ u(2) = -4,\ u(10) = 12 \\ \dfrac{du}{dx} = 2 \end{cases}$$