

MA 114 Worksheet # 11: Density and Average Value

1. Conceptual Understanding:

- (a) If the linear mass density of a rod at position x is given by the function $\rho(x)$, what integral should be evaluated to find the mass of the rod between points a and b ?
- (b) If the radial mass density of a disk centered at the origin is given by the function $\rho(r)$, where r is the distance from the center point, what integral should be evaluated to find the mass of a disk of radius R ?
- (c) Write down the equation for the average value of an integrable function $f(x)$ on $[a, b]$.

2. Find the total mass of a 1-meter rod whose linear density function is $\rho(x) = 10(x + 1)^{-2}$ kg/m for $0 \leq x \leq 2$.

3. Find the average value of the following functions over the given interval.

(a) $f(x) = x^3$, $[0, 4]$

(b) $f(x) = x^3$, $[-1, 1]$

(c) $f(x) = \cos(x)$, $\left[0, \frac{\pi}{6}\right]$

(d) $f(x) = \frac{1}{x^2 + 1}$, $[-1, 1]$

(e) $f(x) = \frac{\sin(\pi/x)}{x^2}$, $[1, 2]$

(f) $f(x) = e^{-nx}$, $[-1, 1]$

(g) $f(x) = 2x^3 - 6x^2$, $[-1, 3]$

(h) $f(x) = x^n$ for $n \geq 0$, $[0, 1]$

4. Odzala National Park in the Republic of the Congo has a high density of gorillas. Suppose that the radial population density is $\rho(r) = 52(1 + r^2)^{-2}$ gorillas per square kilometer, where r is the distance from a grassy clearing with a source of water. Calculate the number of gorillas within a 5 km radius of the clearing.

5. Find the total mass of a circular plate of radius 20 cm whose mass density is the radial function $\rho(r) = 0.03 + 0.01 \cos(\pi r^2)$ g/cm².