MA 114 Worksheet #13: Shell Method and Work

1. Sketch the enclosed region and use the Shell Method to calculate the volume of the solid when rotated about the x-axis.
   (a) $x = \frac{1}{4}y + 1$, $x = 3 - \frac{1}{4}y$, $y = 0$
   (b) $x = y(4 - y)$, $x = 0$

2. Use both the Shell and Disk Methods to calculate the volume obtained by rotating the region under the graph of $f(x) = 8 - x^3$ for $0 \leq x \leq 2$ about:
   (a) the x-axis
   (b) the y-axis

3. Use the Shell method to find the volume obtained by rotating the region bounded by $y = x^2 + 2$, $y = 6$, $x = 0$, and $x = 2$ about the following axes:
   (a) $x = 2$
   (b) $x = -3$

4. Find the integral for the volume of the solid obtained by rotating $f(x) = e^x$ about the y-axis from $0 \leq x \leq 2$. Do not evaluate the integral. This will be done on the next worksheet.

5. Conceptual Understanding:
   (a) Define and describe work. What are its units? What is the difference between work and force?
   (b) Determine the work done in lifting a 1 kg weight through a distance of 1 m near the surface of the earth, maintaining a constant velocity.
   (c) How much work is done in lifting a 1 kg weight up 1 m at a constant velocity and then lowering it back 1 m at a constant velocity?

6. Determine the work done in lifting a 500 kg elevator 1000 m to the top floor of a building. How much work is done lowering a 500 kg elevator 1000 m from the top floor of a building to the ground floor? How much work is done making the round trip?

7. A force of 50 N will stretch a spring from its natural length of 5 cm to 15 cm. How much work will be done in stretching the spring from 15 cm to 30 cm?

8. Calculate the work against gravity required to build a right circular cone of height 4 m and base radius 1.2 m out of a lightweight material of density 600 kg/m^3.

9. Consider a rectangular tank of water that is 5 meters tall and has a base of size 8×4 meters. It has a spout on its top surface. Calculate the work required to pump all of the water out of the tank. Dimensions are in meters, and the density of water is 1000 kg/m^3.