Math 114 Worksheet # 14:
Integration by Parts and Trigonometric Integrals

1. Use the product rule to find $(u(x)v(x))'$. Next use this result to prove integration by parts, namely that $\int u(x)v'(x)dx = u(x)v(x) - \int v(x)u'(x)\,dx$.

2. Which of the following integrals should be solved using substitution and which should be solved using integration by parts?

   (a) $\int x\cos(x^2)\,dx$,

   (b) $\int e^x \sin(x)\,dx$,

   (c) $\int \frac{\ln(\arctan(x))}{1+x^2}dx$,

   (d) $\int xe^{x^2}\,dx$

   Using these examples, try and formulate a general rule for when integration by parts should be used as opposed to substitution.

3. Solve the following integrals using integration by parts:

   (a) $\int x^2\sin(x)\,dx$,

   (b) $\int (2x+1)e^x\,dx$,

   (c) $\int x\sin(3-x)\,dx$,

   (d) $\int 2x\arctan(x)\,dx$,

   (e) $\int \ln(x)\,dx$

4. Prove the reduction formula $\int x^n e^x\,dx = x^n e^x - n\int x^{n-1}e^x\,dx$. Use this to evaluate $\int x^3 e^x dx$.

5. Let $f(x)$ be a twice differentiable function with $f(0) = 6$, $f(1) = 5$, and $f'(1) = 2$. Evaluate $\int_0^1 xf''(x)\,dx$.

6. Evaluate the following integrals.

   (a) $\int \cos^2(x)\,dx$.

   (b) $\int_0^{\pi/2} \sin^2(x)\cos^2(x)\,dx$.

   (c) $\int \sin^3(x)\cos^2(x)\,dx$.

   (d) $\int x^2\cos(x)\,dx$.

   (e) $\int e^x\cos(x)\,dx$.

7. Evaluate $\int \sin(x)\cos(x)\,dx$ by four methods

   (a) the substitution $u = \cos(x)$,

   (b) the substitution $u = \sin(x)$,

   (c) the identity $\sin(2x) = 2\sin(x)\cos(x)$,

   (d) integration by parts.

8. Find the volume of the solid obtained by rotating $f(x) = e^x$ about the y-axis from $0 \le x \le 2$.