MA 114 Worksheet # 16: Review for Exam 2

1. Expand $\dfrac{2}{4-3x}$ into a power series with $c=0$ and determine the radius of convergence.

2. (a) Find the Taylor series centered at zero of the function $f(x)=\ln(x+5)$.

   (b) Find the Taylor series centered at zero of the function $g(x)=x^3\ln(x^2+5)$.

3. Compute $T_3(x)$, the Taylor polynomial of the third order centered at $x=0$, for $f(x)=\cos(x/\pi)$.

4. Compute $T_n(x)$, the Taylor polynomial of the nth order centered at $x=0$, for $f(x)=e^{3x}$.

5. Let $f(x)=e^{-x}$. First compute $T_3(x)$ and then use the error bound to show that $|f(x)-T_3(x)|\le x^4/24$ for all $x\ge 0$.

6. Density and average value:

   (a) Find the total mass of a circular plate of radius 20 cm whose mass density is the radial function $\rho(r)=0.03+0.01\cos(\pi r^2)$ g/cm^2.

   (b) Find the average value of $f(x)=\sin(x)\cos(x)$ over $[0,\pi]$.

7. Volumes:

   (a) (Disks) Let V be the volume of a right circular cone of height 10 whose base is a circle of radius 4. Use similar triangles to find the area of a horizontal cross section at a height y. Using this area, calculate the volume V by integrating the cross-sectional area.

   (b) (Washers) Let R be a region bounded by $y=x^2$ and $y=1$, if R is rotated about x-axis, what is the volume of the resulting solid?

   (c) (Cylindrical Shells) V is obtained by rotating the region under the graph $y=3x^2$ for $0\le x\le 2$ about the y-axis. Calculate the volume of V.

8. Work:
   Calculate the work against gravity required to build a right circular cone of height 4 m and radius 2 m out of a lightweight material of density 600 kg/m^3. (See also question 7(a).)

9. Integration by Parts:

   (a) $\displaystyle\int x^2\cos(x)\,dx$

   (b) $\displaystyle\int 2x\arctan(x)\,dx$

10. Trigonometric Integrals:

    (a) $\displaystyle\int \sin^2(x)\cos^3(x)\,dx$

    (b) $\displaystyle\int \tan^3(x)\sec^3(x)\,dx$