MA 114 Worksheet # 2: Sequences

1. Write the first four terms of the sequences with the following general terms:

   (a) $\dfrac{n!}{2^n}$

   (b) $\dfrac{n}{n+1}$

   (c) $(-1)^{n+1}$

2. Find a formula for the nth term of the sequence $\left\{ \dfrac{1}{1}, -\dfrac{1}{8}, \dfrac{1}{27}, -\dfrac{1}{64}, \dots \right\}$.

3. Conceptual Understanding:

   (a) What is a sequence?

   (b) What does it mean to say that a sequence is bounded?

   (c) What does it mean to say that a sequence is defined recursively?

   (d) What does it mean to say that a sequence converges?

4. Let $a_0 = 0$ and $a_1 = 1$. Write out the first five terms of $\{a_n\}$ where a_n is recursively defined as $a_{n+1} = 3a_{n-1} + a_n{}^2$.

5. Suppose that a sequence $\{a_n\}$ is bounded above and below. Does it converge? If not, produce a counterexample.

6. Show that the sequence with general term $a_n = \dfrac{3n^2}{n^2 + 2}$ is increasing. Find and upper bound. Does $\{a_n\}$ converge?

7. Use the appropriate limit laws and theorems to determine the limit of the sequence or show that it diverges.

   (a) $a_n = 1.01^n$.

   (b) $b_n = \frac{3n^2+n+1}{2n^2-3}$.

   (c) $c_n = e^{1-n^2}$.