

## MA 114 Worksheet # 22: Differential Equations and $y' = k(y - b)$

1. Conceptual Understanding:

- (a) What does it mean to say that a differential equation is first-order (or second-order or third-order...)
- (b) What does it mean to say that a differential equation is linear or nonlinear?

2. Use Separation of Variables to find the general solutions to the following differential equations.

- (a)  $y' + 4xy^2 = 0$
- (b)  $\sqrt{1 - x^2}y' = xy$
- (c)  $(1 + x^2)y' = x^3y$
- (d)  $\sqrt{1 + y^2}y' + \sec x = 0$

3. Solve  $y' = 4y + 24$  subject to the condition that  $y(0) = 5$ .

4. Solve  $y' + 6y = 12$  subject to the condition that  $y(2) = 10$ .

5. Recall that Newton's law of Cooling stipulates that the temperature  $y(t)$  of a cooling object with respect to time satisfies the differential equation

$$y' = -k(y - T_0),$$

where  $k$  is a constant depending on the object and  $T_0$  is the the temperature of the ambient environment. Frank's car engine runs at 210°F. On a 70°F day, he turns off the ignition and notes that five minutes later, the engine has cooled to 160°F.

- (a) Find the cooling constant  $k$ .
  - (b) When will the engine cool to 100°F?
6. A cup of coffee with cooling constant  $k = 0.09\text{min}^{-1}$  is placed in a room of temperature 20°C.
- (a) How quickly is the coffee cooling when the temperature is 80°C?
  - (b) Use the linear approximation to estimate the change in temperature over the next 6 s when the temperature is 80°C.
  - (c) If the coffee is initially served at 90°C, how long will it take to reach an optimal drinking temperature of 65°C?
7. (Extra) A tank has the shape of the parabola  $y = x^2$  revolved about the  $x$ -axis. water leaks from a hole of area  $B = 0.0005 \text{ m}^2$  at the bottom of the tank. Let  $y(t)$  be the water level at time  $t$ . How long does it take for the tank to empty if the initial water level is  $y(0) = 1 \text{ m}$ ?