MA 114 Worksheet # 25: The Logistic Equation and First-Order Linear Equations

1. The population of the world in 1990 was around 5.3 billion. Assume the growth constant is 1/265 and the carrying capacity is 100 billion.
   (a) Write out the logistic model and solve it.
   (b) Use this to estimate the population in 2014 and compare it with the actual population of 7.2 billion.
   (c) Use the logistic model to predict the population in 2100 and 2500.

2. Assume the carrying capacity of the U.S. population is 5 billion.
   (a) Use this and the fact that the population in 1990 was 250 million to find the logistic model for the U.S. population. (Do not solve for k).
   (b) Use the fact that the population in 2000 was 275 million to find k and $P(t)$.
   (c) Predict the U.S. population in 2100 and 2500
   (d) When will the U.S. population reach 350 million?

3. A lake with a carrying capacity of 10,000 fish is stocked with 400 fish. The number of fish triples in the first year.
   (a) Find the logistic model and solve it. (Also find k).
   (b) How long will it take for the population to reach 5000 fish?

4. Let $c > 0$. A differential equation of the form

$$\frac{dy}{dt} = ky^{1+c}$$

   where $k > 0$ is called a *doomsday equation* because $1 + c > 1$.
   (a) Use separation of variables to find the solution of this model with $y(0) = y_0$.
   (b) Show that there is a finite time $t = T$ (doomsday) such that

$$\lim_{t \to T^-} y(t) = \infty.$$

   (c) A certain breed of rabbits has the growth rate term $ky^{1.01}$. Suppose the initial population is 2 and there are 16 rabbits after 3 months. When is doomsday?

5. Consider $y' + x^{-1}y = x^3$.
   (a) Verify that $\alpha(x) = x$ is an integrating factor.
   (b) Show that when multiplied by $\alpha(x)$, the differential equation can be written as $(xy)' = x^4$.
   (c) Conclude that xy is an antiderivative of x^4 and use this information to find the general solution.
   (d) Find the particular solution satisfying $y(1) = 0$.

6. Solve the following differential equations:
   (a) $xy' = y - x$
   (b) $y' + 3x^{-1}y = x + x^{-1}$

7. Solve the following differential equations that satisfy the given initial condition:
   (a) $y' + 3y = e^{2x}$, $y(0) = -1$
   (b) $(\sin x)y' = (\cos x)y + 1$, $y(\pi/4) = 0$