MA 114 Worksheet # 3: Summing an Infinite Series

1. (Review) Compute the following sums

    (a) $\sum_{n=1}^{5} 3n$

    (b) $\sum_{k=3}^{6} \left(\sin \left(\frac{\pi}{2} + \pi k\right) + 2k \right)$

2. Conceptual Understanding:

    (a) What is a series?

    (b) What is the difference between a sequence and a series?

    (c) What does it mean that a series converges?

3. Write the following in summation notation:

    (a) $\frac{1}{9} + \frac{1}{16} + \frac{1}{25} + \frac{1}{36} + \dots$

    (b) $1 - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} + \dots$

4. Calculate S_3, S_4, and S_5 and then find the sum of the telescoping series $S = \sum_{n=1}^{\infty} \left(\frac{1}{n+1} - \frac{1}{n+2} \right)$.

5. Use Theorem 3 of 10.2 (Divergence Test) to prove that the following two series diverge:

    (a) $\sum_{n=1}^{\infty} \frac{n}{10n + 12}$

    (b) $\sum_{n=1}^{\infty} \frac{n}{\sqrt{n^2 + 1}}$

6. Use the formula for the sum of a geometric series to find the sum or state that the series diverges and why:

    (a) $\frac{1}{1} + \frac{1}{8} + \frac{1}{8^2} + \dots$

    (b) $\sum_{n=0}^{\infty} \left(\frac{\pi}{e}\right)^n$.

    (c) $5 - \frac{5}{4} + \frac{5}{4^2} - \frac{5}{4^3} + \dots$

    (d) $\sum_{n=0}^{\infty} \frac{8 + 2^n}{5^n}$.