MA 114 Worksheet # 30:
Final Exam Review

Caution: This review sheet does not cover all the possible problems you may see on the final exam. Be sure to review all topics listed on the course calendar.

1. Integration: Compute each of the following (unless the integral is divergent).
   (a) $\displaystyle\int \frac{\sin(\ln(t))}{t}\, dt$
   (b) $\displaystyle\int e^x \sin x\, dx$
   (c) $\displaystyle\int_0^1 \frac{x-1}{\sqrt{x}}\, dx$
   (d) $\displaystyle\int_0^1 \frac{x-1}{x^2+3x+2}\, dx$
   (e) $\displaystyle\int \frac{1}{x^2\sqrt{25-x^2}}\, dx$
   (f) $\displaystyle\int_{-\infty}^0 xe^x\, dx$
2. How many terms should you take in Simpson's rule to approximate $\displaystyle\int_1^2 (\sin x + x + 1)\, dx$ correct to 5 decimal places?
3. Areas, Volumes, and Lengths: Set up (but do not evaluate) integrals for the following geometric quantities.
   (a) The area enclosed by the curves $y = 1 - 2x^2$, $y = |x|$.
   (b) The volume obtained by rotating the region bounded by the curves $y = 1/x$, $x = 1$, and $x = 5$ about the x-axis.
   (c) The volume obtained by rotating the region bounded by the curves $y = 1/x$, $x = 1$, and $x = 5$ about the y-axis.
   (d) The volume obtained by rotating the region bounded by the curves $y = \tan x$, $y = x$, and $x = \pi/3$ about the y-axis.
   (e) The length of the curve $y = x^2$ from $x = a$ to $x = b$.
   (f) The length of the parametric curve $x = 3t^2$, $y = 2t^3$, $0 \le t \le 2$.
   (g) The area enclosed by the polar curve $r = 1 - \cos\theta$.
4. Parametric equations:
   (a) Consider the curve $x(t) = 3t^2 + t$ and $y(t) = 2t$
      i. Eliminate the parameter, t, to find a Cartesian equation for the curve.
      ii. Find the tangent line to this curve at the point $(x, y) = (14, 4)$.
   (b) Consider the polar curves $r = \sin(2\theta)$ and $r = \cos\theta$.
      i. Determine the Cartesian coordinates (x, y) of the point of intersection which is strictly in the first quadrant, i.e. $x, y > 0$.
      ii. Set up an integral, or integrals, for computing the area of the region in the first quadrant between the bolded portion of the two curves. Do not evaluate the integral(s).

5. Differential Equations

   (a) Solve $\dfrac{dL}{dt} = kL^2 \ln t$, $L(1) = -1$.

   (b) Draw the direction field for the differential equation $y' = y + x$. Sketch the solution which satisfies $y(0) = 0$.

   (c) Find the solution of $y' - y = e^{2x}$, $y(0) = 1$.

   (d) Consider the differential equation $y' = x^2 - y$. If at the n-th iteration of Euler's method with $h = 0.1$ we have $(x_n, y_n) = (3.1, -2)$, what is (x_{n+1}, y_{n+1})?

6. Sequences and Series

   (a) Does the sequence $\left\{ \dfrac{2 + n^3}{4 + 5n^3} \right\}$ converge? If so, what is its limit?

   (b) True or false: If $\lim\limits_{n \to 1} a_n = 0$ then $\sum\limits_{n=0}^{\infty} a_n$ converges?

   (c) Does the series $\sum\limits_{n=1}^{\infty} \dfrac{(-1)^n}{n^{1/3}}$ converge conditionally, converge absolutely, or diverge? Explain.

   (d) $\sum\limits_{n=0}^{\infty} \left(\dfrac{1}{n+1} - \dfrac{1}{n+2} \right)$.

   (e) Test the following series for convergence.

      i. $\sum\limits_{n=2}^{\infty} \dfrac{1}{n\sqrt{\ln n}}$

      ii. $\sum\limits_{n=1}^{\infty} \dfrac{n^7}{7^n}$

      iii. $\sum\limits_{n=0}^{\infty} \dfrac{\cos(n)}{2 + 2^n}$

      iv. $\sum\limits_{n=1}^{\infty} \dfrac{5^n + n^2 + n + 17}{3n^4 + 4^n + 1 + 5}$

7. Power, Maclaurin, and Taylor Series

   (a) Find the Maclaurin series for $\dfrac{x^2}{1+x}$.

   (b) Find the Taylor series for $\cos x$ about $a = \pi/2$.

8. Use the limit comparison test to determine if $\sum\limits_{n=1}^{\infty} \left(1 - \cos \dfrac{1}{n} \right)$ converges or diverges.(Hint: compare with $\sum \dfrac{1}{n^2}$.