MA 114 Worksheet # 7: Power Series

1. Find the radius and interval of convergence for $\sum_{n=0}^{\infty} \frac{(-1)^n n}{4^n}(x-3)^n$.

2. Find the radius and interval of convergence for $4\sum_{n=0}^{\infty} \frac{2^n}{n}(4x-8)^n$.

3. Find the radius and interval of convergence for $\sum_{n=0}^{\infty} \frac{x^{2n}}{(-3)^n}$.

4. Find the radius and interval of convergence for $\sum_{n=0}^{\infty} n!(x-2)^n$.

5. Give the definition of the radius of convergence of a power series $\sum_{n=0}^{\infty} a_n x^n$

6. Use term by term integration and the fact that $\int \frac{1}{1+x^2}\,dx = \arctan(x)$ to derive a power series centered at $x = 0$ for the arctangent function.
   [HINT: $\frac{1}{1+x^2} = \frac{1}{1-(-x^2)}$.]

7. Use the same idea as above to give a series expression for $\ln(1+x)$, given that $\int \frac{dx}{1+x} = \ln(1+x)$. You will again want to manipulate the fraction $\frac{1}{1+x} = \frac{1}{1-(-x)}$ as above.

8. Write $(1+x^2)^{-2}$ as a power series.
   [HINT: Use term by term differentiation.]