

Quiz # 9 — 04/09/15

Answer all questions in a clear and concise manner. Answers that are without explanations or are poorly presented may not receive full credit.

Consider the first-order linear differential equation

$$y' + x^{-1}y = \cos(x^2).$$

a. Show that an integrating factor for this differential equation is $\alpha(x) = x$.

We have that $A(x) = x^{-1}$. So,

$$\alpha(x) = e^{\int A(x)dx} = e^{\ln x} = x.$$

Award 1 point for using the correct formula, and 1 point for the correct calculation.

b. Determine the general solution to this differential equation.

$$\begin{aligned} y &= \frac{1}{\alpha(x)} \left(\int \alpha(x)B(x)dx + C \right), \\ &= \frac{1}{x} \left(\int x \cos(x^2)dx + C \right), \\ &= \frac{1}{x} \left(\frac{1}{2} \sin(x^2) + C \right), \\ &= \frac{\sin(x^2)}{2x} + \frac{C}{x}. \end{aligned}$$

Award one point for setting up the correct integral, and one point for the correct solution.