Name: ______________________________ Section: __________

MA 114 — Calculus II *Spring 2015*

Quiz # 2 — 01/29/15

Answer all questions in a clear and concise manner. Answers that are without explanations or are poorly presented may not receive full credit.

We will show that $\sum_{n=1}^{\infty} \ln\left(\frac{(n+1)^2}{n(n+2)}\right)$ converges by recognizing it as a telescoping series.

Parts **a** and **b** together are worth 1 point. Parts **c**, **d** and **e** are worth 1 point each.

a Show that $\ln\left(\frac{(n+1)^2}{n(n+2)}\right) = \ln\left(\frac{n+1}{n}\right) + \ln\left(\frac{n+1}{n+2}\right)$.

$$\ln\left(\frac{(n+1)^2}{n(n+2)}\right) = \ln\left(\frac{n+1}{n}\cdot\frac{n+1}{n+2}\right) = \ln\left(\frac{n+1}{n}\right) + \ln\left(\frac{n+1}{n+2}\right)$$

b Show that $\ln\left(\frac{n+1}{n}\right) + \ln\left(\frac{n+1}{n+2}\right) = \ln\left(\frac{n+1}{n}\right) - \ln\left(\frac{n+2}{n+1}\right)$.

$$\ln\left(\frac{n+1}{n}\right) + \ln\left(\frac{n+1}{n+2}\right) = \ln\left(\frac{n+1}{n}\right) + \ln\left[\left(\frac{n+2}{n+1}\right)^{-1}\right] = \ln\left(\frac{n+1}{n}\right) - \ln\left(\frac{n+2}{n+1}\right)$$

c Compute and simplify S_2, S_3 and S_4 where S_N denotes the Nth partial sum of the series.

$$S_2 = \ln(2) - \ln\left(\frac{3}{2}\right) + \ln\left(\frac{3}{2}\right) - \ln\left(\frac{4}{3}\right) = \ln(2) - \ln\left(\frac{4}{3}\right)$$

$$S_3 = S_2 + \ln\left(\frac{4}{3}\right) - \ln\left(\frac{5}{4}\right) = \ln(2) - \ln\left(\frac{5}{4}\right)$$

$$S_4 = S_3 + \ln\left(\frac{5}{4}\right) - \ln\left(\frac{6}{5}\right) = \ln(2) - \ln\left(\frac{6}{5}\right)$$

d Give a short and simple version of S_N.

$$S_N = \ln(2) - \ln\left(\frac{N+2}{N+1}\right)$$

e Evaluate $\sum_{n=1}^{\infty} \ln\left(\frac{(n+1)^2}{n(n+2)}\right)$.

$$\sum_{n=1}^{\infty} \ln\left(\frac{(n+1)^2}{n(n+2)}\right) = \lim_{N\to\infty} S_N = \lim_{N\to\infty} \ln(2) - \ln\left(\frac{N+2}{N+1}\right) = \ln(2) + \ln\left(\lim_{N\to\infty}\frac{N+2}{N+1}\right) = \ln(2)$$

Leave a comment if continuity of ln is not mentioned when taking the limit but do not deduct points.