Name: ______________________________ Section: __________

MA 114 — Calculus II *Spring 2015*

Quiz # 3 — 02/05/15

Answer all questions in a clear and concise manner. Answers that are without explanations or are poorly presented may not receive full credit.

1. Determine whether the infinite series $\displaystyle\sum_{n=1}^{\infty} \frac{\cos(3n)}{3^n}$ converges or diverges.

Observe that $0 \leq |\cos(3n)| \leq 1$, so $0 \leq \dfrac{|\cos(3n)|}{3^n} \leq \dfrac{1}{3^n}$.

Now the series $\displaystyle\sum_{n=1}^{\infty} \frac{1}{3^n}$ converges because it is a geometric series with $|r| = \frac{1}{3} < 1$.

By Comparison test we see that $\displaystyle\sum_{n=1}^{\infty} \frac{\cos(3n)}{3^n}$ converges absolutely. Therefore, the series converges.

Award 1 point for comparing it with the correct geometric series and 1 point for drawing the correct conclusion

2. Use the Leibniz alternating series test to see if the following series converges. If it converges, determine whether it converges absolutely or conditionally.

$$\sum_{n=1}^{\infty} \frac{3(-1)^n}{n^2+1}$$

C1: $\displaystyle\lim_{n\to\infty} \frac{3}{n^2+1} = 0$. So the first condition holds.

C2: We need to show that $0 < a_{n+1} \leq a_n$.

$a_n = \frac{3}{n^2+1} > 0$ for all n, so we just need to show that $a_{n+1} \leq a_n$.

Note that $n^2 + 1 < (n+1)^2 + 1$ for all n. So, $\frac{3}{(n+1)^2+1} < \frac{3}{n^2+1}$.

This shows that $0 < a_{n+1} < a_n$ for all n and the second condition holds.

As such, the given series converges by the Leibniz alternating series test.

Next we test for absolute convergence.

Observe that $\left|\frac{3(-1)^n}{n^2+1}\right| = \frac{3}{n^2+1} < \frac{3}{n^2}$.

The series $\sum_{n=1}^{\infty} \frac{1}{n^2}$ converges because it is a p series with $p > 1$.

Therefore, $\sum_{n=1}^{\infty} \frac{3}{n^2}$ also converges and by using the comparison test we see that $\sum_{n=1}^{\infty} \frac{3}{n^2+1}$ also converges.

This implies that the original series $\sum_{n=1}^{\infty} \frac{3(-1)^n}{n^2+1}$ converges absolutely.

Award 1 point for showing convergence and 1 point for showing absolute convergence