Name: ______________________________ Section: ____________

MA 114 — Calculus II *Spring 2015*

Quiz # 4 — 02/19/15

Answer all questions in a clear and concise manner. Answers that are without explanations or are poorly presented may not receive full credit.

1. Find the Maclaurin series for $f(x) = \dfrac{1}{3-2x}$ and find the interval on which the expansion is valid.

Recall that the Maclaurin series for $\frac{1}{1-x} = \sum_{n=0}^{\infty} x^n$.

Then:

$$\begin{aligned} \frac{1}{3-2x} &= \frac{1}{3(1-(\frac{2}{3})x)} \\ &= \frac{1}{3}\sum_{n=0}^{\infty}(\frac{2}{3})^n x^n \end{aligned}$$

Since the expansion for $\frac{1}{1-x}$ is valid for $|x| < 1$ we can conclude that our expansion is valid on $(\frac{-3}{2}, \frac{3}{2})$

Award 1 point for correct series and 1 point for correct interval.

2. Find the Maclaurin series for:

$$f(x) = e^x$$

and find the interval on which the expansion is valid.

First construct a general formula for $f^{(n)}(x)$:

$$f(x) = e^x, f'(x) = e^x, f''(x) = e^x, f'''(x) = e^x, \cdots.$$

Then,

$$f^{(n)}(x) = e^x$$

Therefore:

$$f^{(n)}(0) = 1$$

So the Maclaurin series for $f(x) = e^x$ centered at is $f(x) = \sum_{n=0}^{\infty} \frac{1}{n!} x^n$. We will now use the ratio test to find the valid interval for the expansion:

$$\begin{aligned}
\rho &= \lim_{n\to\infty} \left|\frac{(x)^{n+1}}{(n)!}\frac{(n+1)!}{(x)^n}\right| \\
&= \lim_{n\to\infty} \left|\frac{x}{(n+1)}\right| \\
&= x \lim_{n\to\infty} \left|\frac{1}{(n+1)}\right| \\
&= 0
\end{aligned}$$

So the Maclaurin series is valid on the whole real line.

Award 1 point for finding the Maclaurin Series and 1 point for correct interval. Full points may be awarded if the student memorized the interval and the Maclaurin series.