Name: ____________________ Section: __________

MA 114 — Calculus II *Spring 2015*

Quiz # 7 — 03/26/15

Answer all questions in a clear and concise manner. Answers that are without explanations or are poorly presented may not receive full credit.

Let

$$I = \int \frac{dx}{x^2\sqrt{x^2-2}}.$$

a. Show that the substitution $x = \sqrt{2}\sec\theta$ transforms the integral I into $\frac{1}{2}\int \cos\theta d\theta$.

Let $x = \sqrt{2}\sec\theta$. Then

$$dx = \sqrt{2}\sec\theta\tan\theta d\theta, \qquad \sqrt{x^2-2} = \sqrt{2}\tan\theta.$$

$$I = \int \frac{dx}{x^2\sqrt{x^2-2}} = \int \frac{\sqrt{2}\sec\theta\tan\theta d\theta}{2\sec^2\theta\sqrt{2}\tan\theta} = \frac{1}{2}\int \cos\theta d\theta.$$

Award 1 point for correct values of dx and $\sqrt{x^2-2}$. Also 1 point for correct simplifacitons.

b. Use a right triangle to show that with the above substitution, $\sin\theta = \frac{\sqrt{x^2-2}}{x}$.

The substitution $x = \sec\theta$ implies that the hypotenuse is x and the adjacent edge is 3. Hence

$$\sin\theta = \frac{\text{opposite}}{\text{hypotenuse}} = \frac{\sqrt{x^2-2}}{x}.$$

Award 1 point if the edges of the right triangle are labeled correctly.

c. Evaluate I in terms of x.

$$\begin{aligned} I = \int \frac{dx}{x^2\sqrt{x^2-2}} &= \frac{1}{2}\int \cos\theta d\theta \\ &= \frac{1}{2}\sin\theta + C \\ &= \frac{\sqrt{x^2-2}}{2x} + C. \end{aligned}$$

Award 1 point for correct answer. If student omits the constant, add it without penalty.