

Quiz # 8 — 04/02/15

Answer all questions in a clear and concise manner. Answers that are without explanations or are poorly presented may not receive full credit.

Consider the integral

$$I = \int \frac{(2x^2 + 2x + 3)dx}{(x + 2)(x^2 + 3)}.$$

a. Find the partial fraction decomposition of $\frac{(2x^2+2x+3)}{(x+2)(x^2+3)}$.

$$\frac{(2x^2 + 2x + 3)}{(x + 2)(x^2 + 3)} = \frac{A}{x + 2} + \frac{Bx + C}{x^2 + 3}$$

$$2x^2 + 2x + 3 = A(x^2 + 3) + (Bx + C)(x + 2)$$

Setting $x = -2$ yields:

$$2(-2^2) + 2(-2) + 3 = 7A \implies A = 1.$$

Equating coefficients:

$$x^2(A + B) = 2 \implies (1 + B) = 2 \implies B = 1.$$

$$3A + 2C = 3 \implies 3 + 2C = 3 \implies C = 0.$$

Thus $\frac{(2x^2+2x+3)}{(x+2)(x^2+3)} = \frac{A}{x+2} + \frac{Bx+C}{x^2+3} = \frac{1}{x+2} + \frac{x}{x^2+3}$.

Award 1 point for the correct setup, one point for having at least one correct coefficient, and an additional point for having all coefficients correct.

b. Use the decomposition from (a) to solve the integral I .

$$\begin{aligned} I &= \int \frac{(2x^2 + 2x + 3)dx}{(x + 2)(x^2 + 3)} \\ &= \int \frac{dx}{x + 2} + \int \frac{xdx}{x^2 + 3} \\ &= \ln|x + 2| + \frac{1}{2} \ln|x^2 + 3| + C \end{aligned}$$

Award one point for the correct solution. If the $+C$ is forgotten, add it with no penalty.