Worksheet # 13: Implicit Differentiation

1. Find dy/dx by implicit differentiation.

   (a) $x^3 + y^3 = 1$

   (b) $e^y \cos x = 1 + \sin(xy)$

   (c) $y^2(2 - x) = x^3$

2. Use implicit differentiation to find an equation of the tangent line to the curve at the given point.

   (a) $x^2 + y^2 = x + y - x^3$, $(0, 1)$

   (b) $y^2(y^2 - 4) = x^2(x^2 - 5)$, $(0, -2)$

3. Find the derivative of each of the following.

   (a) $f(x) = \arctan \sqrt{x}$

   (b) $g(x) = \arcsin x^2$

   (c) $h(x) = \arccos(e^{2x})$

   (d) $f(x) = \ln(x^2 + 2)$

   (e) $f(x) = \ln(e^{2x} + 5e^x + 3)$

   (f) $f(x) = \ln(\cos(x))$

4. The equation $x^2 - xy + y^2 = 3$ represents a "rotated" ellipse, that is, an ellipse whose axes are not parallel to the coordinate axes. Find the points where this ellipse crosses the x-axis and show that the tangents at these points are parallel.

5. Prove:

$$\frac{d}{dx} \sec^{-1} x = \frac{1}{x\sqrt{x^2 - 1}}.$$

   [Hint: Use the same technique from the proof for the derivative formula for $\sin^{-1}(x)$. Start by writing $y = \cos^{-1}(x)$ and obtain an expression which can be differentiated implicitly.]