Worksheet # 18: The Mean Value Theorem

1. State the mean value theorem and illustrate the theorem in a sketch.

2. (MA 113 Exam III, Problem 8(c), Spring 2009). Suppose that g is differentiable for all x and that $-5 \leq g'(x) \leq 2$ for all x. Assume also that $g(0) = 2$. Based on this information, is it possible that $g(2) = 8$?

3. Section 4.2 in the text contains the following important corollary which you should commit to memory:

   > **Corollary 7, p. 284:** *If $f'(x) = g'(x)$ for all x in an interval (a, b) then $f(x) = g(x) + c$ for some constant c.*

   Use this result to answer the following questions:

   (a) If $f'(x) = \sin(x)$ and $f(0) = 15$ what is $f(x)$?

   (b) If $f'(x) = \sqrt{x}$ and $f(4) = 5$ what is $f(x)$?

   (c) If $f'(x) = k$ where k is a constant, show that $f(x) = kx + d$ for some other constant d.

4. Verify that the function satisfies the hypotheses of the Mean Value Theorem on the given interval. Then find all numbers c that satisfy the conclusion of the Mean Value Theorem.

   (a) $f(x) = e^{-2x}$, [0,3]

   (b) $f(x) = \dfrac{x}{x+2}$, [1,4]

5. A trucker handed in a ticket at a toll booth showing that in 2 hours she had covered 159 miles on a toll road with speed limit 65 mph. The trucker was cited for speeding. Why?

6. If $f(1) = 10$ and $f'(x) \geq 2$ for $1 \leq x \leq 4$, how small can $f(4)$ possibly be?

7. For what values of a, m, and b does the function

$$f(x) = \begin{cases} 3 & \text{if } x = 0 \\ -x^2 + 3x + a & \text{if } 0 < x < 1 \\ mx + b & \text{if } 1 \leq x \leq 2 \end{cases}$$

   satisfy the hypotheses of the Mean Value Theorem on the interval [0,2]?

8. Determine whether the following statements are true or false. If the statement is false, provide a counterexample.

   (a) If f is differentiable on the open interval (a, b), $f(a) = 1$, and $f(b) = 1$, then $f'(c) = 0$ for some c in (a, b).

   (b) If f is differentiable on the open interval (a, b), continuous on the closed interval $[a, b]$, and $f'(x) \neq 0$ for all x in (a, b), then we have $f(a) \neq f(b)$.

   (c) Suppose f is a continuous function on the closed interval $[a, b]$ and differentiable on the open interval (a, b). If $f(a) = f(b)$, then $f'(\frac{a+b}{2}) = 0$.

   (d) If f is differentiable everywhere and $f(-1) = f(1)$, then there is a number c such that $|c| < 1$ and $f'(c) = 0$.