Worksheet # 26: Definite Integrals

1. (MA 113 Exam IV, Problem 5, Spring 2009). Consider the function $\dfrac{x}{x-1}$.
   (a) Compute the Riemann sum for f on the interval $[2,6]$ with $n=4$ subintervals and the left endpoints as sample points. Give the answer as a rational number (fraction).
   (b) Show that f is decreasing on $[2,6]$.
   (c) Without computing $\displaystyle\int_2^6 f(x)\,dx$, decide whether the Riemann sum in (a) is greater than or less than the integral.

2. Evaluate the following integrals using geometry.
   (a) $\displaystyle\int_0^3 \left(\frac{1}{2}x-1\right)\,dx$
   (b) $\displaystyle\int_{-2}^2 \sqrt{4-x^2}\,dx$
   (c) $\displaystyle\int_0^{10} |x-5|\,dx$

3. Suppose $\displaystyle\int_0^1 f(x)dx = 2$, $\displaystyle\int_1^2 f(x)dx = 3$, $\displaystyle\int_0^1 g(x)dx = -1$, and $\displaystyle\int_0^2 g(x)dx = 4$. Compute the following using the properties of the integral.
   (a) $\displaystyle\int_1^2 g(x)dx$
   (b) $\displaystyle\int_0^2 [2f(x)-3g(x)]dx$
   (c) $\displaystyle\int_1^1 g(x)dx$
   (d) $\displaystyle\int_1^2 f(x)dx + \int_2^0 g(x)dx$
   (e) $\displaystyle\int_0^2 f(x)dx + \int_2^1 g(x)dx$

4. Write the following limits of Riemann sums as definite integrals.
   (a) $\displaystyle\lim_{n\to\infty}\sum_{i=1}^n \frac{i^3}{n^4}$
   (b) $\displaystyle\lim_{n\to\infty}\sum_{i=1}^n \frac{1}{n}\sqrt{3+\frac{i}{n}}$
   (c) $\displaystyle\lim_{n\to\infty}\sum_{i=1}^n \frac{2}{n}\left(2+\frac{2i}{n}\right)^2$

5. Find $\displaystyle\int_0^5 f(x)dx$ if

$$f(x) = \begin{cases} 3 & x < 3 \\ x & x \geq 3 \end{cases}$$