Worksheet # 28: Indefinite Integrals and the Net Change Theorem

1. Compute the definite integral.

   (a) $\int_0^2 4x^5 + x^2 + 2x + 1\ dx$

   (b) $\int_0^{\pi/2} (\sin x + 5\cos x)\ dx$

   (c) $\int_1^{16} \frac{1+\sqrt{x}}{\sqrt{x}}\ dx$

   (d) $\int_1^2 \sqrt{\frac{7}{x^3}}\ dx$

2. Find the general indefinite integral.

   (a) $\int \frac{15}{x} dx$

   (b) $\int \frac{x^2 - \sqrt{x}}{x}\ dx$

   (c) $\int \cos(x) - \sin(x) + e^x\ dx$

   (d) $\int (1 + \tan^2\theta)\ d\theta$

   (e) $\int \sin^2 y\ dy$ [Hint: Use an identity.]

3. Let the velocity of a particle traveling along the x-axis be given by $v(t) = t^2 - 3t + 8$. Find the displacement and distance traveled by the particle from $t = 2$ to $t = 4$ seconds.

4. The velocity of a particle traveling along the x-axis is given by $v(t) = 3t^2 + 8t + 15$ and the particle is initially located 5 m left of the origin. How far does the particle travel from $t = 2$ seconds to $t = 3$ seconds? After 3 seconds where is the particle with respect to the origin?

5. (MA 113 Exam IV, Problem 7, Spring 2009). A particle is traveling along a straight line so that its velocity at time t is given by $v(t) = 4t - t^2$ (measure in meters per second).

   (a) Graph the function $v(t)$.

   (b) Find the total distance traveled by the particle during the time period $0 \leq t \leq 5$.

   (c) Find the net distance traveled by the particle during the time period $0 \leq t \leq 5$.

6. An oil storage tank ruptures and oil leaks from the tank at a rate of $r(t) = 100e^{-0.01t}$ liters per minute. How much oil leaks out during the first hour?

7. (Similar to problem 47, p. 397). Draw the region R that lies between the y-axis and the curve $x = 2y - y^2$ from $y = 0$ to $y = 2$. To find the area between a continuous function f and the x-axis on the interval $[a, b]$, we just evaluate $\int_a^b f(x)\ dx$. Use some intuition to find the area of R.