Worksheet # 30: Review for Exam IV

1. Compute the derivative of the given function.
   (a) $f(\theta) = \cos(2\theta^2 + \theta + 2)$
   (b) $g(u) = \ln(\sin^2 u)$
   (c) $h(x) = \int_{-3599}^{x} t^2 - te^{t^2+t+1}\, dt$
   (d) $r(y) = \arccos(y^3 + 1)$

2. Compute the following definite integrals.
   (a) $\int_{-1}^{1} e^{u+1} du$
   (b) $\int_{-2}^{2} -\sqrt{4 - x^2} dx$
   (c) $\int_{1}^{9} \frac{x - 1}{\sqrt{x}} dx$
   (d) $\int_{0}^{10} |x - 5| dx$
   (e) $\int_{0}^{\pi} \sec^2(t/4) dt$
   (f) $\int_{0}^{1} xe^{-x^2} dx$

3. Provide the most general antiderivative of the following functions.
   (a) $x^4 + x^2 + x + 1000$
   (b) $(3x - 2)^{20}$
   (c) $\frac{\sin(\ln(x))}{x}$

4. Use implicit differentiation to find $\frac{dy}{dx}$.
   (a) $x^2 + xy + y^2 = 16$
   (b) $x^2 + 2xy - y^2 + x = 2$. Also, compute $\frac{dy}{dx}(1, 2)$

5. If $F(x) = \int_{3x^2+1}^{7} \cos t\, dt$ find $F'(x)$. Justify your work carefully.

6. Suppose a bacteria colony grows at a rate of $r(t) = 100 \ln(2)2^t$ with t in hours. By how many bacteria does the population increase from time $t = 1$ to $t = 3$?

7. Use a left Riemann sum with 4 equal subintervals to estimate the value of $\int_{1}^{5} x^2 dx$. Will this estimate be larger or smaller than the actual value of definite integral? Explain.

8. A conical tank with radius 5 m and height 10 m is being filled with water at a rate of 3 m^3 per minute. How fast is the water level increasing when the height is 3?

9. A rectangular storage container with an open top is to have a volume of 10 m^3. The length of the container is twice its width. Material for the base costs \$ 10 per square meter while material for the sides costs \$ 6 per square meter. Find the materials cost for the cheapest possible container.

10. State the mean value theorem. Then if $3 \leq f'(x) \leq 5$ for all x, find the maximum possible value for $f(8) - f(2)$.