Worksheet # 5: Limit Laws

1. Given $\lim_{x \to 2} f(x) = 5$ and $\lim_{x \to 2} g(x) = 2$, use limit laws (justify your work) to compute the following limits. Note when working through a limit problem that your answers should be a chain of equalities. Make sure to keep the $\lim_{x \to a}$ operator until the very last step.

   (a) $\lim_{x \to 2} 2f(x) - g(x)$.

   (b) $\lim_{x \to 2} \dfrac{f(x)g(x)}{x}$.

   (c) $\lim_{x \to 2} f(x)^2 + x \cdot g(x)^2$.

   (d) $\lim_{x \to 2} [f(x)]^{\frac{3}{2}}$.

2. Calculate the following limits if they exist or explain why the limit does not exist.

   (a) $\lim_{x \to 1} \dfrac{x^2 - 1}{x - 1}$

   (b) $\lim_{x \to 1} \dfrac{x^2 - 1}{x - 2}$

   (c) $\lim_{x \to 2^+} \dfrac{x^2 - 1}{x - 2}$

   (d) $\lim_{x \to 9} \dfrac{x - 9}{\sqrt{x} - 3}$

3. Find the value of c such that $\lim_{x \to 2} \dfrac{x^2 + 3x + c}{x - 2}$ exists. What is the limit?

4. Show that $\lim_{h \to 0} \dfrac{|h|}{h}$ does not exist by examining one-sided limits. Then sketch the graph of $\dfrac{|h|}{h}$ and check your reasoning.

5. True or false?

   (a) The direct substitution property can always be used to compute limits.

   (b) Let $f(x) = \dfrac{(x+2)(x-1)}{x-1}$ and $g(x) = x + 2$. Then $f(x) = g(x)$.

   (c) Let $f(x) = \dfrac{(x+2)(x-1)}{x-1}$ and $g(x) = x + 2$. Then $\lim_{x \to 1} f(x) = \lim_{x \to 1} g(x)$.

   (d) If both the one-sided limits of $f(x)$ exist as x approaches a, then $\lim_{x \to a} f(x)$ exists.

   (e) Let $p(x) = c_n x^n + c_{n-1} x^{n-1} + ... + c_1 x + c_0$ be a polynomial with coefficients $c_n, c_{n-1}, ..., c_0$. Then $\lim_{x \to a} p(x) = c_n a^n + c_{n-1} a^{n-1} + ... + c_1 a + c_0$.

   (f) If $\lim_{x \to a} f(x)$ exists then $\lim_{x \to a} f(x) = f(a)$.