Worksheet # 7: Derivatives

1. Comprehension check.

   (a) What does it mean for a function to be continuous at the point a? What does it mean for a function be differentiable at the point a?

   (b) Are differentiable functions also continuous? Are continuous functions also differentiable?

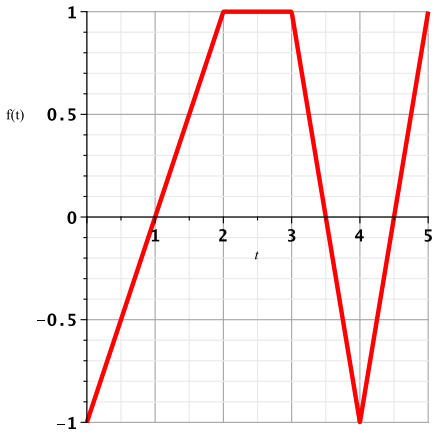
   (c) You have seen three ways in which a function can fail to be differentiable at a point. Sketch these three cases.

   (d) The tangent line to the graph of $g(x)$ at $x = 1$ is given by $y = 5x + 1$. Find $g(1)$ and $g'(1)$.

   (e) Give the two formulas for the definition of the derivative of a function $f(x)$ at a point a.

   (f) What does the derivative of $f(x)$ at $x = a$ describe?

2. A particle is traveling along the x-axis. Below is a graph of its position function $f(t)$ for the time interval $[0, 5]$.



   (a) Graph the particle's velocity function on the time interval $[0, 5]$.

   (b) Graph the particle's acceleration function on the time interval $[0, 5]$.

   (c) For what time intervals is the particle traveling left? Right? When is it at rest?

3. Find $f'(a)$ using either formula of the definition for the derivative:

   (a) $f(x) = 3x^2 - 2x + 1$

   (b) $f(x) = \dfrac{1}{x+3}$

   (c) $f(x) = \sqrt{x}$

4. Use 2(c) to find the tangent line to $f(x) = \sqrt{x}$ when $x = 4$.

5. Let

$$h(t) = \begin{cases} at + b & t \leq 0 \\ t^3 + 1 & t > 0 \end{cases}$$

   Find a and b so that h is differentiable at $t = 0$.