

Worksheet # 8: Review for Exam I

1. Calculate the following limits using the limit laws. Carefully show your work and use only one limit law per step.

(a) $\lim_{x \rightarrow 0} (2x - 1)$

(b) $\lim_{x \rightarrow -1} \frac{x^2 + 1}{x}$

(c) $\lim_{x \rightarrow 1} (3x^3 - 2x^2 + 4)$

2. (a) State the Intermediate Value Theorem.
(b) Use the Intermediate Value Theorem to show that the polynomial $f(x) = x^3 + 2x - 1$ has a zero in some interval of length 1.
(c) Prove that you were once π feet tall.
3. Use the definition of the derivative to find $f'(x)$. Do not use the derivative laws if you know them, because you will not be able to use them on the exam.

(a) $f(x) = \frac{1}{x}$

(b) $f(x) = 3x^2 + 2$

4. (a) State the definition of continuity of a function $f(x)$ at $x = a$
(b) Find the constant a so that the function is continuous on the entire real line.

$$f(x) = \begin{cases} \frac{x^2 - a^2}{x - a} & \text{if } x \neq a \\ 8 & \text{if } x = a \end{cases}$$

5. Let $f(x) = |x|$. From the definitions, prove that $f(x)$ is continuous at $x = 0$ but not differentiable there. Explain how you could surmise this fact from the graph of $f(x)$.
6. The line tangent to the graph of $f(x)$ at $x = 3$ is $y = -2x + 1$. Using this fact, find $f(3)$ and $f'(3)$.