Solutions to Assignment 1 for MA 113 - Calculus I

(1) (5 Points) Consider the functions $f(x) = x + 5$ and $g(x) = 2\sqrt{x+8}$.

(a) Give the domain of the function g.

(b) Sketch the graphs of the functions f and g.

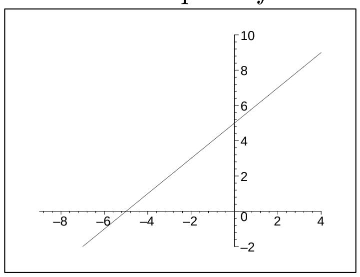
(c) Find the points of intersection of the graphs of f and g.

Solutions:

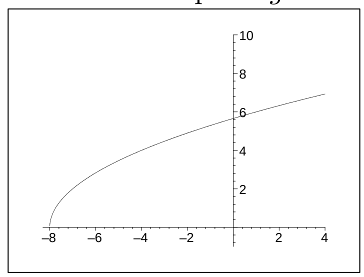
(a) For $g(x)$ to exist we need $x + 8 \geq 0$, thus $x \geq -8$. Hence the domain of g is the interval $[-8, \infty)$.

(b)

Graph of f



Graph of g



(c) We have to find all x such that $f(x) = g(x)$, that is, $x + 5 = 2\sqrt{x+8}$. Squaring both sides we obtain

$$x^2 + 10x + 25 = 4(x + 8),$$

thus

$$x^2 + 6x - 7 = 0.$$

The solutions to this quadratic equation are

$$x_1 = 1, \ x_2 = -7.$$

Hence there are at most two points of intersection, and their x-coordinates are $x_1 = 1$ and $x_2 = -7$. Since $f(1) = 6 = g(1)$ and $f(-7) = -2$, but $g(-7) = 2$, there is just one point of intersection, namely $(1, 6)$.

(2) (5 Points) Consider the function $f(x) = x^2 + 8x + 15$.

(a) Determine the range of f.

(b) Find the largest number a such that f is one-to-one on the interval $(-\infty, a]$ and determine the inverse function g^{-1} of the function $g(x) = f(x)$ with domain $(-\infty, a]$.

(c) Find the smallest number b such that f is one-to-one on the interval $[b, \infty)$ and determine the inverse function h^{-1} of the function $h(x) = f(x)$ with domain $[b, \infty)$.

Solutions:

(a) Completing the square we get $f(x) = (x + 4)^2 - 1$, so the vertex of the parabola $y = f(x)$ is the point $(-4, -1)$. Since the parabola opens up, the range of f is the interval $[-1, \infty)$.

(b) Since the vertex has x-coordinate -4, the function f is increasing on the the interval $[-4, \infty)$ and decreasing on the interval $(-\infty, -4]$. Hence f is one-to-one on the interval $(-\infty, -4]$, but not on any larger interval. Thus, $a = -4$. To determine the inverse function of $g(x) = f(x)$ with domain $(-\infty, -4]$, we have to solve the equation

$$y = x^2 + 8x + 15$$

for x. The quadratic formula applied to $x^2 + 8x + (15 - y) = 0$ provides

$$x = -4 \pm \sqrt{(-4)^2 - (15 - y)} = -4 \pm \sqrt{1 + y}. \tag{1}$$

Since the domain of g is $(-\infty, -4]$, the correct formula for the inverse function is obtained by taking the negative sign. Hence, interchanging x and y, the inverse function of g is

$$g^{-1}(x) = -4 - \sqrt{1+x}$$

with domain $[-1, \infty)$.
(c) As in (b) we conclude that f is one-to-one on the interval $[-4, \infty)$, but not on any larger interval, that is, $b = -4$. To find the inverse function of $h(x) = f(x)$ with domain $[-4, \infty)$, we are led again to Formula (**??**). However, the numbers in the domain of h are at least -4. Hence, we have to use the $+$ sign to get the correct formula for the inverse function, namely

$$h^{-1}(x) = -4 + \sqrt{1+x}.$$

Grading Guidelines:

- Problem (1): (5 points) (a) 1 point. (b) 0.5 points for each graph. (c) 1 point for setting up the equation for x, 1 point for finding x_1 and x_2, 1 point for finding the correct point of intersection.
  Problem (2): (5 points) (a) 1 point. (b) and (c) 1 point for finding a and b, 1 point for each inverse function, and 1 point for explaining the correct choice of the sign.
- Please score in increments of at least 0.5 points. For each part, give full credit only for answers that are both *correct* and *fully explained*.
- Be sure to comment favorably on papers of students who do an unusually good job.
- Take the time to recognize and provide guidance to students who attempt unusual approaches.
- Deductions:
  i) If a student does not use complete sentences, mark with common error "EXP" and ask for complete sentences. Also mark common errors "ALG" and "EQN". Deduct one point for three or more such mistakes which are not otherwise penalized.
  ii) Deduct one point for unusually messy or poorly organized solutions (at most one or two per paper).