Assignment 2 for MA 113 - Calculus I (Spring 2010)

January 28, 2010

Instructions: The purpose of this and subsequent assignments is to develop your ability to formulate and communicate a mathematical argument showing step-by-step reasoning.

Please give a complete, well-written solution to each of the following problems. Your work will be graded for accuracy, completeness, and grammatically correct English.

Your solutions should be neat and legible, stapled, and your name should appear on each sheet. Moreover, on page 1 of your solution, please also indicate your *section number* to insure that you will receive proper credit for the assignment.

Due Date: Your completed solutions are due on **Friday, February 5, 2010**, at the beginning of lecture.

(1) (3 Points) Let x be any real number except -7 and let $f(x) = \dfrac{6}{x+7}$. Find the limit

$$\lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

(the result will be a function of x).

For the following exercise, it will be useful to recall that we can rewrite the difference of radicals by multiplying and dividing by the conjugate:

$$\sqrt{a} - \sqrt{b} = \frac{(\sqrt{a} - \sqrt{b})(\sqrt{a} + \sqrt{b})}{\sqrt{a} + \sqrt{b}} = \frac{a - b}{\sqrt{a} + \sqrt{b}}.$$

(2) (6 Points) Consider the function $f(x) = \sqrt{36 - x^2}$, whose graph describes the upper semicircle with radius 6 centered at the origin. Let $(a, f(a))$ be any point on the graph of $f(x)$, where $a \neq 0$.

(a) Determine the slope of the line through the origin and the point $(a, f(a))$.

(b) Use facts from high school geometry to compute the slope of the tangent line to the semicircle that passes through the point $(a, f(a))$. Clearly state the fact from geometry that you use to compute the slope.

(c) For any number x in $(-6, 6)$, determine the slope of the secant line through the points $(a, f(a))$ and $(x, f(x))$ and compute its limit L as x approaches a. Find the equation of the line through the point $(a, f(a))$ with slope L. From Section 2.1 we know that this line is the tangent line to the graph of f at the point $(a, f(a))$.
Does the slope L coincide with the slope you computed in (b)? If not, go back and try to find your mistake.

(d) Choose a typical number $a \neq 0$ and plot, on the same set of axes, the graph of f, the line through the origin and the point $(a, f(a))$, and the tangent line.

(3) (1 Point) Write the definition of the derivative of a function f at a point a.

Bonus Problem: (2 Points)
Harry and his sister Sally made the following observation about their ages. When you add up Harry's age and Sally's age, the total is 56 years. So, $H + S = 56$. Right now Harry is twice as old as Sally was at the time when he was as old as Sally is now. How old are they?