

## Assignment 5 for MA 113 - Calculus I (Spring 2010)

March 11, 2010

**Instructions:** The purpose of this and subsequent assignments is to develop your ability to formulate and communicate a mathematical argument showing step-by-step reasoning.

Please give a complete, well-written solution to each of the following problems. Your work will be graded for accuracy, completeness, and grammatically correct English.

Your solutions should be neat and legible, stapled, and your name should appear on each sheet. Moreover, on page 1 of your solution, please also indicate your *section number* to ensure that you will receive proper credit for the assignment.

**Due Date:** Your completed solutions are due on **Friday, March 26, 2010**, at the beginning of lecture.

This worksheet will introduce the  $\sum$  (Sigma notation) for summation. This notation will be important later on in Sections 5.1, 5.2 when studying Riemann sums.

Study the pages A 34, A 35, and A 37 of Appendix E and work the following problems.

(1) (2 Points) Write the following sums in expanded form.

$$(a) \sum_{i=1}^7 \sqrt{i+3}. \quad (b) \sum_{i=3}^8 \frac{1}{4i-5}.$$

(2) (3 Points) Write the following sums in Sigma notation.

$$(a) 4 + 6 + 8 + \dots + 22. \quad (b) 5 + 7 + 9 + \dots + 17. \quad (c) 2 + 2x + 2x^2 + 2x^3 + \dots + 2x^n.$$

(3) (2 Points) Find the values of the following sums.

$$(a) \sum_{i=3}^7 (i+2)(i-1). \quad (b) \sum_{i=1}^{80} i.$$

For part (b) you should use one of the formulas presented in the Theorems 2 and 3 of Appendix E. Notice how useful such a short hand formula is. Even with a calculator, adding up the sum in (b) number by number might be a tedious job.

(4) (3 Points)

$$(a) \text{ Verify the identity } \sum_{i=1}^n (3i+1) = \frac{3n^2+5n}{2} \text{ for } n = 4, 5, 6.$$

(b) Use the rules and the formulas given in Theorems 2 and 3 in order to show that

$$\sum_{i=1}^n (3i+1) = \frac{3n^2+5n}{2} \text{ for all natural numbers } n.$$

For the following puzzler you might want to use the answer to one of the previous problems.

**Bonus Problem:** (2 Points)

Somebody is going to recite to you the numbers between 1 and 80, including 1 and 80, but not in the usual order, and she will leave out exactly one number. For instance, the first number she gives you is, say 13, then 4, then 76, then 28 and so on. She is going to leave a space of about 8 seconds between each one. When she finished reciting all the numbers but one, she is going to ask you which one she left out. You are not allowed to use a pencil and paper or other tools, so you can't write anything down or store any data. But you may use the few seconds in between to do, in your head, whatever calculation you may want to. How can you tell which number she left out?