## Solution to Assignment 5 for MA 113 - Calculus I

**(1)** (2 Points) Write the following sums in expanded form.

(a) $\sum\limits_{i=1}^{7} \sqrt{i+3}$. (b) $\sum\limits_{i=3}^{8} \frac{1}{4i-5}$.

**Solution:**

(a) In the expression $\sqrt{i+3}$ we have to substitute for $i$ the numbers from 1 to 7 and add up the results. Thus,

$$\begin{aligned}\sum_{i=1}^{7} \sqrt{i+3} &= \sqrt{1+3} + \sqrt{2+3} + \sqrt{3+3} + \sqrt{4+3} + \sqrt{5+3} + \sqrt{6+3} + \sqrt{7+3} \\ &= 2 + \sqrt{5} + \sqrt{6} + \sqrt{7} + \sqrt{8} + 3 + \sqrt{10}.\end{aligned}$$

(b) This time we have to substitute in the expression $\frac{1}{4i-5}$ for $i$ the numbers from 3 to 8 and add the results.

$$\sum_{i=3}^{8} \frac{1}{4i-5} = \frac{1}{4\cdot3-5} + \frac{1}{4\cdot4-5} + \frac{1}{4\cdot5-5} + \frac{1}{4\cdot6-5} + \frac{1}{4\cdot7-5} + \frac{1}{4\cdot8-5} = \frac{1}{7} + \frac{1}{11} + \frac{1}{15} + \frac{1}{19} + \frac{1}{23} + \frac{1}{27}.$$

**(2)** (3 Points) Write the following sums in Sigma notation.

(a) $4+6+8+\ldots+22$. (b) $5+7+9+\ldots+17$. (c) $2+2x+2x^2+2x^3+\ldots+2x^n$.

**Solution:**

(a) We are adding up the even numbers from 4 to 22. Notice that all even numbers can be expressed as $2i$, where $i$ is any natural number. In particular, the even numbers from 4 to 22 can be expressed as $2i$, where $i$ takes the values from 2 to 11. Thus, we obtain $4+6+8+\ldots+22 = \sum\limits_{i=2}^{11} 2i$.

(b) This time we are adding up the odd numbers from 5 to 17. Since $2i$ is always even, odd numbers can be written as $2i-1$. For $i=3$ this is 5 and for $i=9$ this is 17. The given sum can therefore be written as $5+7+9+\ldots+17 = \sum\limits_{i=3}^{9}(2i-1)$.

(c) We are adding up all the powers of $x$ from exponent 0 to exponent $n$ and multiplied by 2. That can be written as $2+2x+2x^2+2x^3+\ldots+2x^n = \sum\limits_{i=0}^{n} 2x^i$.

**(3)** (2 Points) Find the values of the following sums.

(a) $\sum\limits_{i=3}^{7}(i+2)(i-1)$. (b) $\sum\limits_{i=1}^{80} i$.

For part (b) you should use one of the formulas presented in the Theorems 2 and 3 of Appendix E. Notice how useful such a short hand formula is. Even with a calculator, adding up the sum in (b) number by number might be a tedious job.

**Solution:**

(a) $\sum\limits_{i=3}^{7}(i+2)(i-1) = 5\cdot2 + 6\cdot3 + 7\cdot4 + 8\cdot5 + 9\cdot6 = 150$.

(b) According to Part (c) of Theorem 3 in Appendix E we have $\sum\limits_{i=1}^{n} i = \frac{n(n+1)}{2}$. Using this formula for $n=80$ gives us $\sum\limits_{i=1}^{80} i = \frac{80\cdot81}{2} = 3240$.

**(4)** (3 Points)

(a) Verify the identity $\sum_{i=1}^{n}(3i+1) = \dfrac{3n^2+5n}{2}$ for $n = 4, 5, 6$.

(b) Use the rules and the formulas given in Theorems 2 and 3 in order to show that $\sum_{i=1}^{n}(3i+1) = \dfrac{3n^2+5n}{2}$ for all natural numbers $n$.

**Solution:**

(a) For $n = 4$ the sum is $\sum_{i=1}^{4}(3i+1) = 4 + 7 + 10 + 13 = 34$ and $\frac{3\cdot 16+5\cdot 4}{2} = 34$. Therefore the formula is indeed true for $n = 4$. Likewise one verifies the formula for $n = 5, 6$.

(b) $\sum_{i=1}^{n}(3i+1) \overset{\text{Thm. 2(b)}}{=} \sum_{i=1}^{n}(3i) + \sum_{i=1}^{n} 1 \overset{\text{Thm. 2(a)}}{=} 3\sum_{i=1}^{n} i + \sum_{i=1}^{n} 1 \overset{\text{Thm. 3(a and c)}}{=} 3\frac{n(n+1)}{2} + n \overset{\text{simplify}}{=} \frac{3n^2+5n}{2}$.

**Bonus Problem:** (2 Points)

Somebody is going to recite to you the numbers between 1 and 80, including 1 and 80, but not in the usual order, and she will leave out exactly one number. For instance, the first number she gives you is, say 13, then 4, then 76, then 28 and so on. She is going to leave a space of about 8 seconds between each one. When she finished reciting all the numbers but one, she is going to ask you which one she left out. You are not allowed to use a pencil and paper or other tools, so you can't write anything down or store any data. But you may use the few seconds in between to do, in your head, whatever calculation you may want to. How can you tell which number she left out?

**Solution:** From Problem 3(b) we know that $\sum_{i=1}^{80} i = 3240$. Therefore, if you simply add up all the numbers she is reciting to you, and at the end subtract your count from 3240, the difference is exactly the number she left out.

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**Grading Guidelines:**

- In Problems (1) – (3) each part is worth 1 point.
  In Problem (4) part (a) is worth 1 point and part (b) is worth 2 points.
  Bonus Problem: 1 point for the answer, 1 point for the reasoning.
- Please score in increments of at least 0.5 points. For each part, give full credit only for answers that are both *correct* and *fully explained*.
- Be sure to comment favorably on papers of students who do an unusually good job.
- Take the time to recognize and provide guidance to students who attempt unusual approaches.
- Deductions:
  i) If a student does not use complete sentences, mark with common error "EXP" and ask for complete sentences. Also mark common errors "ALG" and "EQN". Deduct one point for three or more such mistakes which are not otherwise penalized.
  ii) Deduct one point for unusually messy or poorly organized solutions (at most one or two per paper).